STATES OF STRESS IN MULTILAYER SPHERICAL VESSELS, CYLINDRICAL TUBES, AND CIRCULAR DISKS OF LINEAR VISCOELASTIC MATERIAL

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By using the properties of Abel operators and 3-operators [3], unique implicit solutions for N-layer structures with alternating elastic and viscoelastic layers in the case where the viscoelastic operator corresponding to the Poisson ratio reduces to a constant. For an N-layer spherical vessel an analogous implicit solution is also obtained on the assumption that the viscoelastic medium is elastically compressible. Explicit expressions for the reaction pressures are written down for the case of all possible two- and three-layer structures with alternating elastic and viscoelastic layers and analyzed in detail. It is shown that the hypotheses in question lead to qualitatively different states of stress. The general results obtained are illustrated by an example.

In almost all published studies on the analysis of stresses in linear viscoelastic media it is assumed that the medium possesses elastic compressibility or that the viscoelastic operator corresponding to the Poisson ratio reduces to a constant [1]. Since it is not completely clear which of these assumptions is the more realistic, it is necessary to make a joint analysis of the states of stress to which the corresponding assumptions lead.

In this article such an analysis is performed with reference to multilayer spherical vessels, cylindrical tubes, and circular disks. The results can be used in analyzing the state of stress of structurally reinforced spherical vessels, cylindrical tubes, and circular disks of polymeric material and for experimentally testing the strain-hardening hypotheses of the theory of linearly viscoelastic media.

1. Let the multilayer structures in question consist of sets of N interconnected concentric spheres, uniaxial tubes, or disks, in general composed of different materials. These structures are loaded by uniform internal and external pressure, which may slowly vary with time, and are subjected to stationary or slowly varying heating.

In the general case, the multilayer tubes and disks are assumed to rotate at a constant or slowly varying angular velocity. It is also assumed that all the elements of the tube and the disk are under conditions of plane strain and plane stress, respectively.

We introduce the nondimensional quantities: 

\[ r_i = \frac{r_i}{l}; \quad T_i = \frac{T_i^*}{T^*}; \quad E_i = \frac{E_i^*}{E^*}; \quad \sigma_{ii} = \frac{\sigma_{ii}^*}{\sigma^*}; \quad \alpha_i = \frac{\alpha_i}{l}; \quad \alpha_{N+i} = \frac{\alpha_{N+i}}{l}; \]

\[ \rho_i = \frac{\rho_i^*}{\rho}; \quad \rho_{N+i} = \frac{\rho_{N+i}^*}{\rho}; \quad \rho_{N+i}^* \quad \text{are functions of temperature} \quad (i = 1, 2, \ldots, N); \]

\[ T_i^* \quad \text{are functions of time} \quad (i = 1, 2, \ldots, N); \]

\[ E_i^* \quad \text{are the Young moduli, Poisson ratios, and linear expansion coefficients}; \]

\[ \sigma_{ii}^*, \quad \rho_{N+i}^* \quad \text{are the component stresses and displacements}; \]

\[ \alpha_i \quad \text{are the inside radii}; \quad b, \quad p_i, \quad p_{N+i}^* \quad \text{are the outside radius and the internal and external pressure for the multilayer structures in question}; \]

\[ \rho_i, \rho_{N+i} \quad \text{are the reaction pressures; \quad } \rho_i(a_i \leq \rho_i \leq a_{N+i}) \quad \text{are the variable radii}; \]

\[ \tau^* \quad \text{is variable time}; \quad \gamma_1 \quad \text{are densities}; \quad \omega \quad \text{is the angular velocity}. \]

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If all the elements of the structures are elastic, then, as in [2], we obtain the following expressions for $\sigma_{ri}, \sigma_{0i}, \nu_i$:

$$\sigma_{ri} = \sum_{k=1}^{3} C_{ki} \frac{\kappa_{ik}}{r_i};$$  

(1.1)

$$\sigma_{0i} = \frac{1}{q} \sum_{k=1}^{3} C_{ki} \left[ \kappa_{ik} + (q-1) \frac{\kappa_{ik}}{r_i} \right] + (2-q) \omega^* r_i^2;$$

(1.2)

$$u_i = \frac{r_i}{E'q} \left[ \sum_{k=1}^{3} C_{ki} \kappa_{ik} + (2-q) \omega^* r_i^2 \right] + \beta_i r_i T_i(r_i),$$

(1.3)

where

$$A_{ki} = \kappa_{ki} [1 - (q-1) \nu^* i] + \kappa_{ki} [(q-1)(1-\nu^* i) - q\nu^* i];$$

$$C_{ij} = (D_{ij} + B_{ij}) D_i^{-1},$$

$$C_{21} = -(D_{21} + B_{21}) D_i^{-1},$$

$$C_{31} = 1;$$

$$D_i = \kappa_{2i}(\alpha_i) \kappa_{1i}(\alpha_m) - \kappa_{2i}(\alpha_m) \kappa_{1i}(\alpha_i);$$

$$B_i = \kappa_{3i}(\alpha_i) \kappa_{1i}(\alpha_m) - \kappa_{3i}(\alpha_m) \kappa_{1i}(\alpha_i);$$

$$B_{2i} = \kappa_{3i}(\alpha_i) \kappa_{1i}(\alpha_m) - \kappa_{3i}(\alpha_m) \kappa_{1i}(\alpha_i);$$

$$m = i + 1.$$

A prime denotes the derivative with respect to $r_i$.

The reaction pressures are expressed in terms of recurrence relations:

$$y_j = (K_{j-1} - d_{2j} \nu_{j+1}) I_{j-1}^{-1} \quad (j = 2, 3, \ldots, N);$$

(1.4)

$$K_0 = p_i; \quad K_{j-1} = M_{j-1} - d_{1j} K_{j-2} I_{j-2}^{-1}.$$

In these expressions $J_0 = J_1 = 1;

$$J_j = 1 - d_{1j} d_{2j} \nu_{j-1} I_{j-1}^{-1} \quad (j = 2, 3, \ldots, N - 1);$$

(1.5)

$$d_{1j} = H_{j} S_{j-1}, \quad d_{2j} = H_{2j} S_{j-1};$$

(1.6)

$$H_{1i} = \alpha_i [A_{1i}(\alpha_m) \kappa_{2i}(\alpha_m) - A_{1i}(\alpha_m) \kappa_{2i}(\alpha_m)] D_m;$$

$$H_{2i} = \alpha_{m+1}[A_{1m}(\alpha_m) \kappa_{2m}(\alpha_m) - A_{2m}(\alpha_m) \kappa_{1m}(\alpha_m)] D_m;$$

$$R_i = D_i r_i I_m(\alpha_m) - L_i(\alpha_m) D_m;$$

$$L_i(r_i) = B_{1i} A_{1i}(r_i) - B_{2i} A_{2i}(r_i) + D_i A_{3i}(r_i) + \beta_i^* T_i(r_i) D_i E_i^* q + D_i (2-q) \omega^* r_i^2;$$

$$S_i = \alpha_m [D_m A_{2i}(\alpha_m) \kappa_{1i}(\alpha_m) - A_{1i}(\alpha_m) \kappa_{2i}(\alpha_m)] + n_{1i} D_i [A_{2m}(\alpha_m) \kappa_{1m}(\alpha_m) - A_{1m}(\alpha_m) \kappa_{2m}(\alpha_m)]$$

$$n_i = E_i^* (E_i^* m)^{-1}; \quad m = i + 1; \quad \kappa_{ij} = r_i; \quad \kappa_{2i} = r_i - q;$$

$$X_{2i} = -\frac{1}{8} (2-q) (3+v^* i) \omega^* r_i^2 \beta_i^* r_i^q \Gamma(1+\alpha);$$

$$\nu_{x_i} = \nu_{v_i};$$

(2.1)

$$\nu_{x_i} = \nu_{v_i};$$

In the case of a disk and a sphere:

$$E_i^* = E_i, \quad \nu_i^* = \nu_i, \quad \beta_i^* = \beta_i.$$

In the case of a tube:

$$E_i^* = \frac{E_i}{1-\nu_i^*}, \quad \nu_i^* = \frac{\nu_i}{1-\nu_i};$$

$$\beta_i^* = (1+\nu_i) \beta_i.$$  

In the case of a sphere $q = 2$, in the case of a tube and a disk $q = 1.$

Substituting for the elastic constants $\nu_i, E_i$ in expressions (1.3), (1.4) the elastic operators $\tilde{\nu}_i, \tilde{E}_i$ and interpreting the expressions obtained, we obtain the solution of the corresponding problem in the case of elements composed of a linear viscoelastic material [3].

2. Let us consider the case where elastic and viscoelastic layers alternate, assuming the absence of temperature stresses and rotation. Let the subscript $t$ correspond to the elastic and the subscript $s$ to the viscoelastic layer. As the creep kernel we take the Abel kernel [3]:

$$I_2(\tau - \tau^*) = \frac{(\tau - \tau^*)^a}{\Gamma(1+\alpha)}; \quad -1 < \alpha < 0.$$

(Here $\tau^*$ is the relaxation time, $\Gamma(1+\alpha)$ is the gamma function.)

If it is assumed [1] that

$$\tilde{\nu}_s = \nu_s;$$

(2.1)