METHODS OF EXPLOITING NATURAL INTERNAL STRESSES TO STRENGTHEN COMPOSITES

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A method of strengthening nonuniformly reinforced composites is proposed. A rational scheme for coordinating the external stress field, the resistance field, and the internal stress field is examined in relation to the case when the internal stresses are caused by shrinkage of the resin.

When a composite of the reinforced-plastic type polymerized at an elevated temperature is cooled to the operating temperature, temperature stresses develop \[1\]. In the case of cold polymerization corresponding shrinkage stresses are observed. These stresses cannot be eliminated completely. Accordingly, to distinguish them from the artificial internal stresses created, for example, by prestressing the reinforcement, we shall call them "natural."

If the internal stresses have the same sign as the stresses created by the external load, they reduce the carrying capacity of the structure; if, however, the stresses are of opposite sign, then the net stresses are less than nominal and, consequently, the carrying capacity will be greater than nominal (i.e., greater than the carrying capacity of the material without internal stresses). In uniformly reinforced composites natural internal stresses usually have an adverse effect on the performance of the material: they reduce its strength, cause warping, etc. Accordingly, they are undesirable, and the problem of reducing them is one of the more urgent problems of polymer physicochemistry and mechanics. Obviously, in this connection, it is no less desirable to develop a means of utilizing natural internal stresses for reinforcement purposes.

In order to make the natural internal stresses not only harmless but also useful, it is necessary to coordinate three principal fields – those of the stresses due to the external load, the resistances (moduli of elasticity, yield points, strength, etc.), and the internal stresses. An example of the coordination of the first two fields is given in \[2\]. Below we examine a method of coordinating all three fields in cases where the internal stresses are created by shrinkage of the resin. The method is based on the extension of the statistical boundary value problem of the theory of elasticity of macrohomogeneous two-component continua with shrinkage stresses \[3\] to include nonhomogeneous media. The problem is solved by generalizing Melan's method previously proposed for homogeneous media \[4\].

The results may be of practical importance not only in relation to improvements in carrying capacity but also as a means of rationalizing the selection of resins. It is known that materials that are expensive or in short supply are often used simply because they have relatively low shrinkage. If the shrinkage stresses are put to good use, then, other things being equal, the material with greater shrinkage will be more suitable. Consequently, it becomes unnecessary to employ expensive and scarce resins.

1. The basic system of equations of the statistical boundary value problem of the theory of elasticity of a two-component continuum having internal stresses due to shrinkage of the resin is written \[3\]:

\[
D \cdot \sigma = 0; \quad \varepsilon = -\frac{1}{2} (D_N + \tau \xi) \quad \sigma = \Theta \cdot (\varepsilon - b \lambda).
\]  

(1.1)

Here, $\sigma(x)$, $\varepsilon(x)$, $\chi(x)$, $\Theta(x)$ are the random stress, strain, displacement, and elastic modulus tensors; $x$ is the radius vector; $b$ is the shrinkage strain tensor; $\lambda(x)$ is a stochastic indicator function: $\lambda = 1$ if an element belongs to the resin and $\lambda = 0$ if it belong to the reinforcement; $D = \int_{\Omega}(\partial/\partial x_{i})\partial \sigma \cdot \Theta \cdots (\varepsilon - b\lambda)$ are scalar and scalar triple products; $\Delta x$ and $\chi D$ are formal (dyadic) products.

If the medium is macrohomogeneous and $\Theta = C + \Delta$; $\lambda = P + \omega$ ($C = \langle \Theta \rangle$; $P = \langle \lambda \rangle$) is the symbol of mathematical expectation ME), then $C = \text{const}$; $P = \text{const}$ with respect to $x$. The method of solving system (1.1) in this case was previously described in [3].

Let the medium be macroinhomogeneous. Then $\Theta = C_x + \Delta(x)$; $\lambda = P_x + \omega(x)$; $C_x = C(x)$; $P_x = P(x)$. We set

$$C_v = \frac{1}{V} \int C_x dV; \quad P_v = \frac{1}{V} \int P_x dV,$$

where $V$ is a region of the body with boundary $S$. Moreover, $C_x = C_v + C_b(x)$; $P_x = P_v + P_b(x)$; $\omega = \varepsilon + \rho$; $\chi = \mu + \tau$; $\rho = \langle \omega \rangle$; $\varepsilon = \varepsilon + \eta$; $\mu = \mu + \zeta$; $\eta = \eta + \zeta$. Using this notation, we transform (1.1):

$$C_v \cdot D \cdot \varepsilon \cdot D = -D \cdot \Pi \cdot \varepsilon (1.2)$$

Equation (1.2) with the boundary condition $\varepsilon |_{S} = \zeta(\varepsilon)(x)$ is equivalent to the integral equation

$$\eta = T \cdot \Pi \cdot \varepsilon \cdot q (1.3)$$

Here, $T \cdot \Pi = \frac{1}{2} [T_1 \cdot \Pi + (T_1 \cdot \Pi) D]$; $T_1 \cdot \Pi = \int G(x, x') \cdot (\Pi' \cdot D') dV'$; $G(x, x')$ is the Green's tensor of the region $V$; $q = \frac{1}{2} (Dq + q D)$; $q_{1} = \int G'(x, x') \cdot C_v \cdot \zeta' \cdot n'dS'$; $G'(x, x') = DG(x, x')$; $n$ is the unit vector of the outward normal to the boundary $S$ of the region $V$.

Solving Eq. (1.3) by an iterative method, we find

$$\eta = (\Lambda^* \cdot \varepsilon) + \beta - \tau + \varepsilon (1.4)$$

Here, we have introduced the notation: $(\Lambda^* \cdot \varepsilon) = \Lambda^* \cdot \varepsilon + \Lambda^* \cdot \varepsilon'' + \cdots$; $\beta = F + \Lambda^* \cdot F' + \Lambda^* \cdot F'' + \cdots$; $\tau = \tau_0 + \Lambda^* \cdot \tau'_0 + \Lambda^* \cdot \tau''_0 + \cdots$; $\varepsilon = \varepsilon^1 + \Lambda^* \cdot \varepsilon'' + \Lambda^* \cdot \varepsilon''' + \cdots$; $F = T \cdot (C_v \cdot \varepsilon')$; $\tau_0 = T \cdot (\Theta \cdot b\lambda')$; $\Lambda^* \cdot \varepsilon'' = T \cdot ((\varepsilon^1 + \Delta') \cdot \varepsilon)$.

2. We apply the ME operator to (1.1):

$$D \cdot p = 0; \quad \varepsilon = \frac{1}{2} (Du + uD);$$

$$p = C_x \cdot \cdot (e + b P_x) + \langle \Delta \cdot \eta + b\omega \rangle (2.1)$$

In projection on the coordinate axes, system of Eqs. (2.1) comprises 15 equations. If the reinforcement distribution (relative volume reinforcement content $Q_x = 1 - P_x$) is given, then system (2.1) contains 21 unknown functions. Accordingly, it is not closed. The extra unknown functions are the components of the tensor $(\Delta^*(\eta + b\omega))$.

In order to close system (2.1), we eliminate the superfluous unknown functions by applying Eqs. (1.4). Here, we shall confine ourselves to the case when the random displacements on the boundary $S$ of the region $V$ are statistically independent of the physical properties of the interior points. The extension to dependent boundary conditions is obvious.

Assuming that $C^I$ and $C^{II}$ are the deterministic tensors of the elastic moduli of the resin and the reinforcement, $C^* = C^I - C^{II}$, and using the obvious equations $C_x = C^I P_x + C^{II} Q_x$; $\Delta = C^* \omega$, we find

$$p = C_x \cdot \cdot e + C^* \cdot \cdot \langle \Theta (\Lambda^* \cdot \varepsilon) \rangle + C_x \cdot \cdot b P_x + C^* \cdot \cdot \langle \omega (\beta - \tau + b\omega) \rangle (2.2)$$

Together with the first two of Eqs. (2.1), Eq. (2.2) forms a closed system of equations analogous to (but not physically identical with) the system of equations of the boundary value problem of the classical theory of elasticity.