Il'yushin's method of approximations is used to obtain the exact solutions of problems of viscoelasticity from the known solutions of the corresponding elastic problems for a concentrated force acting at a point in space, a parallelepiped with one loaded face, a rotating annular disk, and a gravitating rotating sphere (the earth). The solutions of the viscoelastic problems are determined by three functions — the compliance, relaxation modules, and "connected" creep functions. A description is given of a method and testing device for determining the "connected" creep functions and the corresponding experimental curves for polymethyl metacrylate are presented.

In [1, 2] Il'yushin proposed an effective method of solving boundary value problems of viscoelasticity, according to which the solution of any static problem can be represented in the form

\[ \tilde{S} = f(x, t) + \int_0^t \left[ \phi(t-s) \frac{\partial \Phi(x, s)}{\partial s} + \chi(t-s) \frac{\partial \Phi(x, s)}{\partial s} + g_{\beta n}(t-s) \frac{\partial \Phi(x, s)}{\partial s} \right] ds, \] (1)

where \( \tilde{S} \) is an unknown quantity, for example, a strain, displacement or stress; \( \phi(t) = R(t/3B) \) is the relaxation kernel; \( \chi(t) = 3BH(t) \) is the creep kernel; \( B \) is the bulk modulus, \( f, \phi, \chi, x \) are certain coefficients that do not depend on Poisson's ratio and are defined in terms of loads, displacements, and \( x \) coordinates given in the body and on its surface; \( g_{\beta n}(t) \) is a certain kernel whose Laplace-Carson transformation gives the operator

\[ g^*_{\beta n} = \frac{1}{1 + \beta_n \omega} ; \] (2)

\( \beta_n \) are constant parameters that depend on \( \omega \). The functions \( \phi(t) \) and \( \chi(t) \) are determined from relaxation and creep tests. The determination of the function \( g_{\beta n}(t) \) is investigated in [1-3], where it is proposed to approximate the operators \( g^*_{\beta n} \) with polynomials and construct \( g_{\beta n}(t) \) on the basis of tests conducted in a high-pressure chamber in accordance with a special program of variation of the tensile load and pressure with time [3] and on the basis of a creep test with an included elastic element [2].

Here, the methods of [1, 2] are used to construct the exact solutions of problems of viscoelasticity on the assumption that the bulk modulus is constant. The elastic solution of the static problem at constant temperature can be written for the displacements \( U \) and stresses \( S \) in the form

\[ U = \mathcal{W} \tilde{k}_{11}(r, v) + \frac{P}{E} \tilde{k}_{12}(r, v); \quad S = E \mathcal{W} \tilde{k}_{21}(r, v) + P \tilde{k}_{22}(r, v), \] (3)

where \( \tilde{k}_{ij}(r, v) \) are functions of the coordinates, dimensions and Poisson's ratio \( v \); \( E \) is Young's modulus; \( \mathcal{W} \) are certain quantities such as displacements, deflection, curvature, etc., whose dimensionality depends
only on length; \( P \) is a quantity measured in units of force. Solution (3) can be written in the form

\[
U = \omega W k_{11} (r, \omega) + \frac{P}{2G} \tilde{k}_{12} (r, \omega); \quad S = 2G \omega W k_{21} (r, \omega) + P k_{22} (r, \omega)
\]

or

\[
U = \omega W k_{11} (r, \omega) + \frac{P}{3B} k_{12} (r, \omega); \quad S = 3B \omega W k_{21} (r, \omega) + P k_{22} (r, \omega),
\]

which are obtained from (3) by making the substitutions

\[
\begin{align*}
\nu &= \frac{1 - \omega}{2 + \omega}; & E &= 2G (1 - \nu) = \frac{6G}{2 + \omega} = \frac{9B \omega}{2 + \omega}; & \omega &= \frac{1 - 2\nu}{1 + \nu}; \\
\tilde{\omega} &= \omega k_{12}; \quad \omega \tilde{k}_{21} = k_{21}.
\end{align*}
\]

The functions \( k_{ij} (r, \omega) \) are known quantities that can be represented in the form

\[
k_{ij} (r, \omega) = \frac{1}{\omega} k_{ij}^{(-1)} (r) + k_{ij}^{(0)} (r) + \omega k_{ij}^{(1)} (r) + \frac{1}{1 + \beta \omega} k_{ij}^{(0)} \quad (i, j = 1, 2).
\]

Substituting (6) in (4), we obtain the solution of the elastic problem in the form

\[
U = \frac{1}{\omega} \left[ W k_{11}^{(-1)} (r) + \frac{P}{3B} k_{12}^{(-1)} (r) \right] + \left[ W k_{11}^{(0)} (r) + \frac{P}{3B} k_{12}^{(0)} (r) \right] \\
+ \omega \left[ W k_{11}^{(1)} (r) + \frac{P}{3B} k_{12}^{(1)} (r) \right] + \frac{1}{1 + \beta \omega} \left[ W k_{11}^{(0)} (r) + \frac{P}{3B} k_{12}^{(0)} (r) \right];
\]

\[
S = \frac{1}{\omega} \left[ 3B W k_{21}^{(-1)} (r) + P k_{22}^{(-1)} (r) \right] + \left[ 3B W k_{21}^{(0)} (r) + P k_{22}^{(0)} (r) \right] \\
+ \omega \left[ 3B W k_{21}^{(1)} (r) + P k_{22}^{(1)} (r) \right] + \frac{1}{1 + \beta \omega} \left[ 3B W k_{21}^{(0)} (r) + P k_{22}^{(0)} (r) \right].
\]

The solution of the viscoelastic problem in Laplace–Carson transforms has the form of the solution of the elastic problem written in the transforms. Making in (4) (or (7)) the formal substitutions

\[
\begin{align*}
\frac{1}{\omega} &\rightarrow 3B \Pi^*; & \omega &\rightarrow \frac{R^*}{3B}; & \frac{1}{1 + \beta \omega} &\rightarrow g^*_\beta; & 2G &\rightarrow R^*; \\
\frac{1}{2G} &\rightarrow \Pi^*; & P &\rightarrow P^*; & S &\rightarrow S^*; & W &\rightarrow W^*; & U &\rightarrow U^*; & B &\rightarrow B,
\end{align*}
\]

we obtain:

\[
U^* = k_{11}^* W^* + k_{12}^* \frac{P^*}{3B}; \quad S^* = k_{11}^* 3B W^* + k_{22}^* P^*;
\]

\[
k_{ij}^* (P) = k_{ij}^{(-1)} (r) 3B \Pi^* + k_{ij}^{(0)} (r) W^* + k_{ij}^{(1)} (r) \frac{R^*}{3B} + k_{ij}^{(0)} (r) g^*_\beta W^* + k_{ij}^{(1)} (r) g^*_\beta P^*;
\]

or in expanded form

\[
U^* = k_{11}^{(-1)} (r) 3B \Pi^* W^* + k_{11}^{(0)} (r) W^* + k_{11}^{(1)} (r) \frac{R^*}{3B} + k_{11}^{(0)} (r) g^*_\beta W^* \\
+ k_{12}^{(-1)} (r) \Pi^* P^* + k_{12}^{(0)} (r) \frac{1}{3B} P^* + k_{12}^{(1)} (r) \frac{1}{9B^2} R^* P^* + k_{12}^{(0)} (r) \frac{1}{3B} g^*_\beta P^*;
\]