Formulas are given for calculating the coefficients of differential operators of defining equations on the basis of given approximations of the relaxation kernels in the form of the sum of exponential curves. As the defining equations it is suggested to use quadrature formulas into which are substituted the relaxation kernels found experimentally without preliminary analytic approximation. A three-dimensional difference problem of the linear isotropic theory of viscoelasticity is formulated. The direct and inverse $\xi$-transformation establishing the correspondence between the viscoelastic and elastic difference problems is introduced. The specific characteristics of the use of the net, Ritz, finite-element, and variation-difference methods in solving problems of viscoelasticity theory are examined. A method facilitating the arrangement of the information on relaxation kernels in a computer memory is indicated.

1. The number of investigators in the field of mechanics of a solid deformable body using numerical methods increases with each year. This is related first of all with progress in computing machinery and with the increasing complexity of problems being raised by practice that are to be solved. Numerical methods are the most effective approximate methods by means of which the most diverse problems of mechanics are solved. However, it is very difficult to answer the question of wherein lies the difference between an exact and an approximate solution of a particular problem. The author knows of cases where an "exact" analytic solution of the problem of calculating the numerical values met with insurmountable difficulties and was therefore replaced by an approximate numerical solution. Most often in practice investigators run into the situation where it is necessary to obtain numerical values of the solution of a certain problem with a certain given accuracy by the least laborious method.

The most widespread numerical method is the net method, according to which a table of approximate values of the solution being sought is found for some set of points, called a net, the individual points being called mesh points.

At present there are outstanding guides on the net method (for example, [1-4]) and a large literature on the application of these methods to problems of continuum mechanics. However, it should be noted that there are considerably fewer works on the application of the net method to problems of mechanics of a solid deformable body in which an estimate of the accuracy of the solution obtained is given. Such problems are rather complex and for their solution require a large internal storage and much computer time. According to the classification given in [5], problems for the solution of which 10 mesh points per measurement ($N = 10$) are required are called rough problems (A), problems for which $N = 10^2$ are called problems of average accuracy (B), and problems with $N = 10^3$ are called exact problems (C). If $n$ is the number of measurements of the problem being considered, the computer time for a BESM-4 computer in solving a problem of viscoelasticity theory for 100 time steps can be determined from Table 1 (below). We borrowed this table (with appropriate conversion) from [5]. We see from it that the net method is far from all-powerful.

In this article we will introduce the $\xi$-transformation of mesh functions, which is useful in many cases, and will examine the specific characteristics of the application of numerical methods — net, projection-difference, and variation-difference — to the solution of problems of viscoelasticity theory.
2. Defining Equations. The equations of motion of continuum mechanics have the form

\[
\sigma_{ij,j} + \rho F_i = \rho u_i',
\]

(2.1)

where \(\sigma_{ij}\) is the symmetric stress tensor; the \(\rho F_i\) are body forces; \(u_i\) is the displacement vector; \(\rho\) is the density of the substance. The comma, as usual, denotes covariant differentiation and the dot on the upper right of the expression denotes the time derivative of this expression. We will consider strains \(\varepsilon_{ij}\) to be small:

\[
\varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}).
\]

(2.2)

To close the complete set of equations of the isothermal theory of viscoelasticity additional relationships between stresses and strains are written in the form of integral time operators. For the linear isotropic theory of viscoelasticity these relationships have the form

\[
s_{ij} = \int_0^t \Gamma(t - \tau) e_{ij}(\tau) d\tau = \tilde{\Gamma} e_{ij}; \quad \sigma_0 = \int_0^t \Gamma_1(t - \tau) \theta(\tau) d\tau = \tilde{\Gamma}_1 \theta,
\]

(2.3)

where \(s_{ij}\), \(e_{ij}\) are respectively the stress and strain deviators; \(\sigma_0 = 1/3 c_{kk}\); \(\theta = \varepsilon_{ii}\). Substituting relationships (2.3) into (2.1) and using (2.2), we obtain a set of three integro-differential equations in the displacement vector \(u_i\):

\[
\tilde{L}(u) + \rho F = \rho u',
\]

(2.4)

here we should add some boundary conditions

\[
|u| \bigg|_{t=0} = N_0,
\]

(2.5)

for example

\[
u_i \bigg|_{x=} = u_i^0; \quad \sigma_i \bigg|_{x=} = S_i^0
\]

(2.5')

and the initial data.

Differential time operators are sometime used as the relation between stresses and strains. Thus, it was proved in [6] that any nondegenerate model composed of springs and dashpots reduces to a differential operator between stresses \(\sigma\) and \(\varepsilon\):

\[
P\sigma = Q\varepsilon,
\]

(2.6)

where \(P = \sum_{i=0}^m a_i d^i; \quad Q = \sum_{i=0}^n b_i d^i\) \((a_m = 1, \quad b^n \neq 0)\). Here \(d\) is the operator of differentiation with respect to time \(d \equiv d/dt; \quad d f = d f/dt = f^*\). The numbers \(m\) and \(n\) are called the orders of the operators \(P\) and \(Q\), respectively. As is known, there are only two possibilities: \(n = m\) or \(n = m + 1\) [7]. We will call the bodies for which \(n = m + 1\) liquid and the bodies for which \(n = m\) solid. Relationships (2.6) for the viscoelastic models can be written in an integral form:

\[
\sigma(t) - \int_0^t \Gamma(t - \tau) e(\tau) d\tau = \tilde{\Gamma} e,
\]

(2.7)