DEFORMATIVE PROPERTIES OF FLEXIBLE PLASTIC FOAMS IN COMPRESSION

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A calculation of the compression diagrams of flexible plastic foams during their long-time testing under a load is given on the basis of the proposed cell model, on the assumption that permanent set is related to buckling of the strands. The character of change of the nominal compressive strength and corresponding deformation as a function of the magnitude of permanent set is established. A marked change of the second section of the compression diagram (plateau), even to its degeneration, in the presence of considerable permanent set was found. A comparison of the experimental and calculated data showed their satisfactory correspondence.

During prolonged use of flexible plastic foams in a loaded state a change is observed in the physical and mechanical properties of the foam, characterized by a decrease of its rigidity, occurrence of permanent set, and change of the form of the deformation curves. The change of the cellular structure occurring in this case should have a specific effect on the mechanical properties of flexible foams, since the parameters of the macrostructure have a determining effect on the character of deformation of cellular polymeric materials [1-4]. Therefore, the purpose of this investigation was to study the relation between the deformative properties of flexible foams and change of the macrostructure during their long-time stay in a loaded state.

Presented below are the results of an analytic calculation based on a proposed cell model of the dependence of the properties of flexible foam on the parameters of the cellular structure in compression tests, since compressive loads are most characteristic for the service conditions of these materials. To simplify the calculation we will consider the behavior of an isotropic foam with an initial ratio of the longitudinal dimensions of the cells to the transverse equal to unity. The cell model used for the calculation is described in [3]. Strands (bars) connected into squares form the cellular structure of this type. All adjacent squares are connected together at the corners only in mutually perpendicular planes. In cooperative compression, most energetically advantageous bending of the strands occurs, which is equivalent to the work of strands according to a scheme with one clamped end.

We introduce the following notation: \( b \) is the initial length of the strand; \( D \) is the width of the strand; \( E \) is the Young modulus of the polymer-base; \( I = D^4/12 \) is the moment of inertia of the section of the strand; \( \beta = D/b \) is a constant; \( P \) is the force compressing the strand in longitudinal bending; \( P_{OC} \) is the force compressing the cell; \( \varepsilon \) is the unit compressive strain of the specimen; \( \varepsilon_{cr} \) is the unit strain of the specimen at the nominal compressive stress (critical strain). It corresponds to a marked change of rigidity of the specimen and is measured at the intersection of the tangents at the place of the first inflection of the compression diagram; \( \varepsilon_{per} \) is the unit permanent set of the specimen; \( \varepsilon' = \varepsilon_{per} + \varepsilon \) is an introduced notation; \( \gamma \) is the apparent density of the initial foam (before load testing); \( \rho \) is the density of the polymer base; \( \sigma_{cr} \) is the nominal compressive stress of the initial foam. It is determined by the compressive stress at which a marked change of rigidity of the specimen is observed and is measured at the intersection of the tangents at the place of the first inflection of the compression diagram; \( \sigma_{cr} \) is the nominal compressive strength of the foam after load testing; \( \sigma \) is the compressive strength of the foam;


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Fig. 1. Diagram of the work of the strand after load testing of plastic foam: Solid line) strand in an unloaded state; L) end point of the strand.

$\sigma_{ch}$ is a correction taking into account the increase of stress due to crushing of the strands on the end section of the compression diagram; $p = b \sqrt{P/EI}$ is a force parameter; $1/R$ is the initial curvature of the strand (after load testing of the foam); $m$ is the modulus of an elliptic integral; $\psi_L$ is the table amplitude of the elliptic integral; $\xi = b/R$ is an introduced notation; $S$ is the length of the arc of the strand reckoned from the start of the strand 0.

Direct observations showed that the change of the cellular structure consists mainly in buckling of the load-bearing elements of the macrostructure (load-bearing strands) and in their preservation in a bent state. We will consider that the initial curvature of the strand is constant over its length. In this case we will estimate the large displacements of the strands, using the method of elastic parameters [5, 6].

The differential equation of the elastic line of the strand (Fig. 1) has the form

$$b^2 \frac{d^2 \xi}{dS^2} = -p \sin \xi,$$

where $\xi$ is the slope angle of the tangent to the x axis.

In the initial section of the compression diagram the force $P$ is small and the elastic line is without an inflection. Point L in this case is not a point of inflection, and the curvature here is equal to the initial curvature $1/R$. Then the solution of Eq. (1) for the end point L gives

$$x_L = \frac{2}{mp} E(\psi_L) - \frac{2}{m^2} + 1;$$

$$y_L = \frac{2}{mp} \left(1 - \sqrt{1 - m^2 \sin^2 \psi_L}\right),$$

whereupon

$$\frac{b}{R} = \frac{2p}{m} \sqrt{1 - m^2 \sin^2 \psi_L};$$

$$p = mF(\psi_L).$$

Here

$$E(\psi_L) = \int_0^{\psi_L} \sqrt{1 - m^2 \sin^2 \psi} \, d\psi;$$

$$F(\psi_L) = \int_0^{\psi_L} \frac{d\psi}{\sqrt{1 - m^2 \sin^2 \psi}}.$$ 

To find the relations between the force parameter $p$ and the quantities $x_L$ and $y_L$ we can use first of all Eqs. (3) and (4), from which we easily obtain

$$2F(\psi_L) \sqrt{1 - m^2 \sin^2 \psi_L} = \frac{b}{R}.$$

In Eq. (5) we select for individual values of $m$ the corresponding values $\psi_L$ by means of tables [7]. Then on the basis of the known values of $m$ and $\psi_L$ we find by means of Eqs. (2) and (4) the values of $p$, $x_L$, and $y_L$.

Upon an increase of deflection of the strand the modulus of the elliptic integral passes through the value $m = 1$, which in the given case is related to transition from the uninflected to the inflected form of equilibrium. Since there are no tables of elliptic integrals for $m > 1$, further calculations are reorganized so that the modulus $m$ remains, as before, less than unity. Then we obtain

$$\frac{b}{R} = 2mp \cos \psi_L.$$