EFFECT OF LOADING HISTORY ON CHANGES IN THE CHARACTERISTIC ANGLES AND THE LENGTH OF THE RETARDATION TRACE

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The local strains theory and the theory of small elastoplastic strains are used to determine the values of the components of the plastic strain vector \( \varepsilon(\vartheta_1, \vartheta_2) \) for nonlinearity \( n = 3 \) in five-dimensional Euclidean space in the case of complex loading along a two-segment broken line, when both tensile and compressive stresses are present in each loading stage. The relation between the vectors \( \varepsilon \) and \( S \) and the tangents to the deformation and loading trajectories are examined. Values of the retardation trace are obtained in terms of the loading history. Numerical results have been derived with the aid of a BESM-3M computer.

It is assumed that an incompressible plastic material isotropic and homogeneous in the starting state is subjected to complex loading in accordance with a piecewise-linear law; loading is effected in two stages (Fig. 1). In both stages compressive stresses \( \sigma_{II} \) and tensile stresses \( \sigma_{22} \) act jointly. For a plane state of stress the loading paths considered are characterized by the parameters \( k_1, k_2, k, \eta, \) and \( \xi, \) which depend on the angle \( \delta = 180^\circ + \eta - \lambda \) (\( \delta \) is the angle between the stress vectors of the first and second loading stages in the plane \(-\sigma_{II}, \sigma_{22}\) and is assumed positive when reckoned in the counterclockwise direction).

In the general case: \( k_1 = \tan \eta = |\sigma_{II}*|/|\sigma_{jj}*| \); \( k_2 = \tan \lambda = |(\sigma_{II}-\sigma_{II}*)/|\sigma_{jj} - \sigma_{jj}*| | \) (where \( k_1 \) and \( k_2 \) = const for each loading history); \( x = \tan \eta \); \( k = k_1 = k_{II} = \tan (\eta + \lambda) = |\sigma_{II}/\sigma_{jj}| = (k_1 + x)/(1 - k_1 x) \) (here and in what follows the subscript \( I \) relates to calculations in accordance with the local strains theory and a subscript \( II \) to calculations in accordance with the theory of small elastoplastic strains); \( n = (\sigma_{jj} - \sigma_{jj}*)/\sigma_{jj}* = (k_1 - k)/(k - k_2) \) (where \( i = 1, j = 2 \) at \( \delta > 0^\circ \), and \( i = 2 \) and \( j = 1 \) at \( \delta < 0^\circ \)).

We have investigated loading paths with the following values of the parameters \( k_1 \) and \( k_2 \) at \(-180^\circ < \delta < 180^\circ \); \( k_1 = 0, \sqrt{3}/3, 1, \sqrt{3}, \infty \); \( k_2 = 0, \sqrt{3}/3, 1, \sqrt{3}, \infty \), i.e., 20 cases of complex loading.

Calculation of the Strain Components \( \varepsilon(\vartheta_1, \vartheta_2) \) in Accordance with the Local Strains Theory [1]

We assume that the incremental loading condition [2] for the second loading stage is given by the inequality

\[
\tau_{II} > \tau_{II*} \cos (\tau_{II}, \tau_{II*})
\]

or in expanded form

\[
\sigma_{12}^I + \sigma_{21}^I > \sigma_{12}^{I*} + \sigma_{21}^{I*} + \sigma_{T1}^I \sigma_{T2}^I,
\]

where the local shear stresses \( (\beta^I = 1^I, 2^I) \), which depend on the state of stress given by the stress tensor \( \sigma_{kl} \) \( (k, l = 1, 2) \) are equal to \( \sigma_{12}^I = -\sigma_{13}^I l_{13}^I \beta^I + \sigma_{23}^I l_{23}^I \beta^I \). Here, \( l_{13}^I = \cos \theta, l_{23}^I = \cos \varphi \sin \theta \) are direction cosines.

After transformation condition (2) may be written in the following form


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Clearly, for the given loading paths condition (3) is satisfied for any value of k, since k is always greater than zero, i.e., the plastic strains increase at all values of the stresses of the second loading stage. After integration of the local strain component $\varepsilon_{3j}'$ in accordance with the relation

$$ e_{ij}^{pl} = \frac{1}{S} \int_S e_{ij} \cdot l_{ij} \cdot l_{ij} \cdot ds $$

and further transformations, we obtain the following values for the components of the plastic strain vector $\varepsilon(\delta, \Theta)$ in five-dimensional space. The calculation procedure is described in [3].

For $0^\circ < \delta < 180^\circ$ we have

$$ \Theta_1 = \sqrt{\frac{3}{2}} A \left\{ C_3 \left( -2 - \frac{2}{k_2} - \frac{3}{4k_2^2} \right) + C_2 \left( -3 - \frac{9}{4k_2} + \frac{3k_1}{k_2} + \frac{9k_1}{k_2^2} \right) \right. $$

$$ + C_1 \left[ -\frac{9}{4} \left( 1 - \frac{k_1}{k_2} \right)^2 n \left( C_1 + \frac{27}{8} + \frac{9}{4} n \right) \right] + \left[ (1+n) \right] \left[ \left( 1+n \right) \right] \}

$$

Similarly, $\Theta_2$ is obtained for $0^\circ > \delta > -180^\circ$.

Calculation of the Components of the Strain Vector $\Theta(\delta, \Theta)$ in Accordance with the Theory of Small Elastoplastic Strains [4]

According to this theory