Network polymers and the corresponding glass-reinforced plastics are investigated for a homogeneous uniaxial state of stress and constant temperature. A physical law relating the high-elastic strain and the stress in explicit form with once-determined structural constants is obtained for the damped (bounded) creep regime. The theoretical solutions are compared with the experimental data for a glass-reinforced plastic based on unsaturated polyester resin and glass mat reinforcement.

In investigating the relation between stresses, strains, and time for viscoelastic media most authors adopt a phenomenological approach [1-5]. In this case the rheological constants entering into the equations are correspondingly determined. A physically based relation is proposed in [6]. In this physical law the material characteristics are employed; however, it is complicated, since it relates the stresses with the strains in implicit form.

We have set ourselves the task of simplifying Rabinovich's physical equation [6] for the case of bounded creep, i.e., of finding a sufficiently accurate physical relation that gives the stress-strain dependence in explicit form and includes the material characteristics. Network polymers and the corresponding glass-reinforced plastics are studied. The uniaxial state of stress is investigated for the damped (bounded) creep regime at constant temperature.

Following [7, 8], we write the refined phenomenological law of deformation of polymers and glass-reinforced plastics with allowance for elastic and high-elastic strains in the form

\[ nH + Ee = \alpha + n\dot{e}. \]  

(1)

Here, \( E = E_\infty H / (E_\infty + H) \) is the long-time modulus of elasticity; \( H \) is the instantaneous modulus of elasticity; \( n = k / (E_\infty + H) \) is the relaxation time coefficient; \( E_\infty \) is the high-elastic modulus; \( k \) is the viscosity coefficient

\[ k = k_0 [1 - k_A \exp (-\lambda t)]. \]  

(2)

where \( k_0 \), \( k_A \), and \( \lambda \) are determined with the aim of accurately describing the experimentally obtained creep curves.

In [7, 8] expression (2) is used for correcting the relaxation time coefficient \( n(t) \) in the form

\[ n(t) = n_0 [1 - l_A \exp (-\lambda t)]. \]  

(3)

Equation (2) coincides with (3) correct to a constant.

Equation (1) corresponds to the rheological model of a "standard solid" with variable viscosity coefficient \( k \) (Fig. 1).
On the other hand, following [6], at constant temperature for the loading regime in question a physical relation between stress and strain can be written in the form

\[
\frac{d\sigma}{dt} = \frac{1}{H} \frac{d\sigma}{dt} + \frac{\sigma - \sigma_0}{\eta_0} \exp \left\{ \left[ \frac{1}{3} \gamma^* \sigma + \left| \sigma - \sigma_0 \right| \right] \frac{1}{m^*} \right\},
\]

where \( \nu_e \) is the relative strain rate; \( \sigma \) is the normal stress; \( \lambda \) is the instantaneous modulus of elasticity; \( E_\infty \) is the high-elastic modulus; \( m^* \) is the logarithmic rate modulus; \( \eta_0^* \) is the initial viscosity coefficient; \( \epsilon \) is the total strain; \( \gamma^* \) is the volume coefficient. Since for the creep regime in question the expression \( \sigma - \sigma_0 \) is always positive, there is no need for the sign of the absolute quantity. Equation (4) is based on the generalized Maxwell–Gurevich rheological model, which, if the residual strain is neglected, coincides with the model of a "standard solid" (see Fig. 1). Consequently, equation (4) can be represented by equation (1) by expressing the viscosity coefficient in the latter in the following form:

\[
k = \eta_0^* \exp \left[ \frac{E_\infty \gamma^* - \sigma}{m^*} - \frac{\gamma^* \sigma}{3m^*} \right].
\]

We solve the problem we have formulated by using the rheological model shown in Fig. 1 and described by Eq. (1) with viscosity coefficient (2). In this case the rheological constants \( k_0, k_A, \) and \( \chi \) are found after equating (2) to (5) with allowance for the conditions that follow from the damped creep regime:

\[
\eta_0^* \exp \left[ \frac{E_\infty \gamma^* - \sigma}{m^*} - \frac{\gamma^* \sigma}{3m^*} \right] = \frac{E_\infty \gamma^* - \sigma}{m^*} - \frac{\gamma^* \sigma}{3m^*},
\]

with the conditions \( t = 0 \rightarrow \epsilon^* = 0 \) and \( t \rightarrow \infty \rightarrow \epsilon^* = \sigma/E_\infty \) we obtain

\[
k_0 = \eta_0^* \exp \left( -\frac{\gamma^* \sigma}{3m^*} \right); \quad k_A = 1 - \exp \left( -\frac{\sigma}{m^*} \right).
\]

The coefficient \( \chi \) is determined as follows: we set \( \frac{E_\infty}{k_0} = \alpha = \text{const} \). The high-elastic deformation of the model shown in Fig. 1 is described by a first-order linear differential equation

\[
\sigma = E_\infty \gamma^* + k_0 [1 - k_A \exp (-\chi t)] \epsilon^*.
\]

The solution of Eq. (8) with the initial condition \( t = 0 \rightarrow \epsilon^* = 0 \) is obtained in the form

\[
\epsilon^* = \frac{\sigma}{E_\infty} \left[ 1 - \frac{(1 - k_A)^{\alpha/\chi}}{(e^{\chi t} - k_A)^{\alpha/\chi}} \right]
\]

The first derivative of the high-elastic strain (8)

\[
\dot{\epsilon}^* = \frac{\sigma \alpha}{E_\infty} \frac{(1 - k_A)^{\alpha/\chi}}{\chi \left( e^{\chi t} - k_A \right)^{\alpha/\chi}}.
\]

After differentiating (6) once with respect to time we obtain

\[
k_0 k_A \chi \exp (-\chi t) = \eta_0^* \left[ \frac{E_\infty}{m^*} \exp \left( \frac{E_\infty \gamma^* - \sigma}{m^*} - \frac{\gamma^* \sigma}{3m^*} \right) \right] \dot{\epsilon}^*.
\]