We study an approximate method for the analytic determination of steady temperature fields in the elements of optical systems. The method assumed can be used to study temperature fields of other objects with a curvilinear boundary.

Optical systems often undergo the action of various energetic factors. This leads to the appearance of inhomogeneous temperature fields in the system and thermoelastic stresses in its separate elements (lenses, illuminators, mirrors). The presence of stresses produces deformations in optical elements, changes their form, and the parameters of the system are different from those calculated. This is expressed as a transform of thermooptical aberrations. In this connection we are most interested in the initial determination of the effect of energetic factors on the quality of the optical system operation in order to further use this data in the planning stage for developing compensation systems or automatic control. In this scheme the calculation of the temperature field is the first stage which imposes definite requirements on the solution, notably the sufficient accuracy and the relative simplicity of the final result. Below we study an approximate analytic method to determine steady temperature fields in optical elements which satisfies these requirements to a sufficient degree.

General Formulation of the Problem

We study a lens whose surfaces $S_1$, $S_2$, and $S_3$ are located in three media with the different temperatures $t_{c1}$, $t_{c2}$, and $t_{c3}$ (Fig. 1). We assume that the heat exchange of the surfaces with the media is realized according to Newton's law with the constant coefficients of heat exchange $\alpha_1$, $\alpha_2$, and $\alpha_3$. On the lens surface we give the heat flows with surface density $q_1(s_1)$, $q_2(s_2)$, $q_3(s_3)$ that can be a coordinate function in the general case. Internal energy sources with volume power $w(v)$ can also act on the lens. The absorbed part of the falling flows can play the role of the sources. Both the convex and the concave refracting surfaces $S_1$ and $S_2$ in the cylindrical coordinate system are described by the equations $z(s_1) = f_1(r)$, $z(s_2) = f_2(r)$. To solve the problem we assume that the physical parameters are constant. Assuming axial symmetry, we write the mathematical notation of the problem formulated as follows:

\[
\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial z^2} + \frac{w(r, z)}{\lambda} = 0, \tag{1}
\]

\[
\left[ \frac{\partial t}{\partial n_1} + \frac{\alpha_1}{\lambda} (t - t_{c1}) \right]_{z = f_1} = \frac{q_1(r)}{\lambda}, \tag{2a}
\]

\[
\left[ \frac{\partial t}{\partial n_2} + \frac{\alpha_2}{\lambda} (t - t_{c2}) \right]_{z = f_2} = \frac{q_2(r)}{\lambda}, \tag{2b}
\]

\[
\left[ \frac{\partial t}{\partial r} + \frac{\alpha_3}{\lambda} (t - t_{c3}) \right]_{r = R} = \frac{q_3(z)}{\lambda}. \tag{2c}
\]

We study the case of constant energetic actions

\[q_1(r) = q_{10}; \quad q_2(r) = q_{20}; \quad q_3(r) = q_{30}; \quad w(r, z) = w_0.\]

This limitation is made to illustrate the method assumed more clearly and has no principal value.

We write the temperature derivative along the normal which enters boundary conditions (2a) and (2b) by means of the equations
\[
\frac{\partial t}{\partial n} = \frac{\partial t}{\partial n} \cos(n, z) - \frac{\partial t}{\partial r} \cos(n, r).
\] (3)

Using the axial thickness of the lens \(d\) and the radius \(R\), we transfer to the relative coordinates \(\rho = r/R, \zeta = z/d\). Here the parameter \(\kappa = d^2/R^2\) appears in problem (1)-(2). This quantity is usually small for most lenses, which allows us to use the excitation method [1] and select \(\kappa\) as the small parameter \(\varepsilon\). We also note that in optics the main interest is the superheating \(\theta = t - t_c\), with respect to which problem (1)-(2) takes the following form with the small parameter taken into account:
\[
\frac{\partial^2 \theta}{\partial \rho^2} + \frac{\varepsilon \partial^2 \theta}{\partial \rho^2} + \frac{\varepsilon^2 \partial^2 \theta}{\partial \rho^2} + \frac{\varepsilon \partial \theta}{\partial \rho} = 0,
\] (4)
\[
\left[ \varepsilon \frac{\partial \theta}{\partial \rho} + \frac{\partial \theta}{\partial \zeta} + B_{ij} \sqrt{1 + \varepsilon (f_{ij})^2} \right]_{t = f_1} = Q_{10} \frac{1}{1 + \varepsilon (f_{ij})^2},
\] (5a)
\[
\left[ -\varepsilon \frac{\partial \theta}{\partial \rho} + \frac{\partial \theta}{\partial \zeta} + B_{ij} \sqrt{1 + \varepsilon (f_{ij})^2} \right]_{t = f_2} = Q_{20} \frac{1}{1 + \varepsilon (f_{ij})^2},
\] (5b)
\[
\left[ \varepsilon \frac{\partial \theta}{\partial \rho} + B_{ij} \theta \right]_{\rho = f_3} = Q_{30}.
\] (5c)

Here
\[
f_1 = \frac{f_1}{d}; f_2 = \frac{f_2}{d}; B_{ij} = \frac{\sigma d}{\lambda}, i, j = 1, 2, 3.
\]

The quantities
\[
Q_{10} = \frac{q_{10} d}{\lambda} + (t_{c1} - t_{c3}) B_{1i}, Q_{20} = \frac{q_{20} d}{\lambda} + (t_{c2} - t_{c3}) B_{2i}, Q_{30} = \frac{q_{30} d}{\lambda}
\]
can be studied as generalized flows.

The linear formulation of the problem allows us to study the effect of each of the energetic actions \(w_0, Q_{10}, Q_{20}, Q_{30}\) separately. Here the desired temperature field is obtained by means of the simple summation of the separate solutions.

**Internal Energy Source**

The temperature field of the lens with an internal energy source is described by Eq. (4) under homogeneous boundary conditions (5). In accordance with the excitation method we present the expansion of the desired function \(\theta_0\) in the central (with respect to \(\rho\)) region as follows [1]:

![Fig. 1. Thermal model of problem.](image-url)