MULTIPARAMETRIC PREDICTION OF THE CREEP OF POLYMER MATERIALS.

REPORT 1

Yu. S. Urzhumtsev and R. D. Maksimov

The article discusses phenomenological methods for predicting long-term creep, using stress-, temperature-, and vibro-time analogies. Starting from several previously introduced hypotheses, the article proposes a generalized nonlinear thermo-vibro-rheological relationship in which the complex stress-, temperature-, and vibro-time dependences are reduced separately to a time dependence (with certain basic reference values of \( \sigma_{10}, T_0, \) and \( \dot{\sigma}_{klq} \), chosen arbitrarily), and to \( (\sigma, T, \dot{\sigma}_{klq})^{-1} \) dependences; the latter appears only in the change of the time factors \( a_\sigma(\sigma), a_\sigma(T), \) and \( a_\sigma(\dot{\sigma}_{klq}) \). The prediction methods have been verified experimentally.

The prediction of the long-term creep of polymer materials from the results of short-term laboratory tests is based mainly on the method of the temperature-time analogy [1-3]. However, this method has narrow limits of applicability; it is limited to a determined temperature and time range, to a linear character of the viscoelastic properties, and to simple types of loading.

At the same time, under real conditions, polymer structures work under the simultaneous action of different temperature conditions and high static loads, prolonged by the action of additional vibrational loading. Under these conditions, the material is, as a rule, in a complex state of stress; this can be attained either by simple or by unproportional loading.

An increase in the stresses in materials with an unclearly marked transition to nonlinearity leads to a change in the relaxation time [1] and, consequently, to a corresponding shift of the creep curves along the time scale [4]. The imposition of vibrations on constant loads brings about a vibro-creep, due both to heating up of the material as the result of hysteresis losses, and to an acceleration of the relaxation processes connected with the specific vibrational action [5-7].

In the present work the problem was posed of developing phenomenological methods of predicting creep, based on the use of only a few parameters, taking account of the effect of temperature, stresses, and vibrational loading. Before proceeding to an examination of these methods, it will be useful to make a few general remarks on the hypotheses which have been introduced.

HYPOTHESIS I (Temperature-Time Analogy)*. If the material, at a temperature \( T = T_0 \), has a continuous relaxation spectrum \( LT_0(\tau) \), in the time interval between \( \tau_0 \) and \( \tau_0 + d\tau_0 \), at \( T = T_1 \) the relaxation process may be replaced by a process with a relaxation time between \( \tau_0 / \varphi(T_1) \) and \( (\tau_0 + d\tau_0) / \varphi(T_1) \). At the same time, the characteristics of the elasticity do not change since, in accordance with the hypothesis adopted \( LT_1(\tau)d\tau = \varphi(T_1)LT_0(\varphi(T_1)\tau)d\tau \) at \( \tau = \tau_0 / \varphi(T_1) \) reduces to \( LT_0(\tau_0)d\tau_0 \), where \( \int \limits_0^\infty LT(\tau)d\tau = I(\infty) \) is the equilibrium pliability of the material.

*Hypothesis I was introduced by Shinozuka [8].
Fig. 1. Shift of the relaxation spectrum with a change in the temperature, the imposition of a vibrational load and an increase in the intensity of the stresses, $\sigma_1 = \sigma_{i_1}(1), \sigma_{i_0}(2-4)$; $T = T_1(1, 2), T_0(3, 4)$; $\delta a = \delta \sigma_1(1-3), 0(4)$. $\tau_3 = \tau_1/a \sigma_1(\delta \sigma_1); \tau_4 = \tau_2/a \sigma_1(\delta \sigma_1); \tau_5 = \tau_3/a \sigma_1(T_1 - T_0); \tau_6 = \tau_4/a \sigma_1(T_1 - T_0); \tau_7 = \tau_5/a \sigma_1(\sigma_{i_1} - \sigma_{i_0}); \tau_8 = \tau_6/a \sigma_1(\sigma_{i_1} - \sigma_{i_0})$.

The geometric meaning of these relationships comes down to the fact that, with a change in the temperature, the elements of area and the total area, limited by functions of the density of the spectrum and by the time scale (the area of the spectrum), remain constant. Only the density of the spectrum and the relaxation time scale are transformed. Under these circumstances, the transformation under consideration is merely a hyperbolic rotation of the plane [9].

Formally, using a Dirac $\delta$-function, the discrete relaxation spectrum can be included in the function $L_\tau(\tau)$. It follows from Hypothesis I that if, at $T = T_0$, the material has a discrete relaxation spectrum $(I_1; \tau_i)$, the spectrum at $T = T_1$ is $[I_1; \tau_1/\varphi(T_1)]$.

Essentially, such relationships were established in [10], starting from dimensional considerations. These relationships also constitute the basis of the method of the temperature-time analogy.

If, as a rheological model, we use an equation for the viscoelasticity which includes a continuous distribution of the relaxation spectrum, the relationship for the thermal creep of an isotropic material assumes the form:

$$I_{ij}\Delta(t, T) = b_T \int_{-\infty}^{+\infty} L_{ij}(\ln \tau, T_0) \left(1 - e^{-\frac{\tau}{\tau_0}}\right) d\ln \tau,$$

where $a_T$ is the temperature-time coefficient; $b_T$ is the vertical *shear factor* (usually $b_T = \rho_0 T_0 / \rho T$; $\rho$ is the density). Here, for all level of $T$, exactly the same parameters of the spectrum $I(\ln \tau, T_0)$ are taken; this flows out of Hypothesis I.

HYPOTHESIS II (Stress-Time Analogy). If a nonlinear viscoelastic material without a sharply expressed transition to nonlinearity, with a "basic" value of the intensity of the stresses $\sigma_{i_0}$, and with a given constant temperature $T_0$, has a relaxation spectrum $L_{\sigma_0}(\tau)$, the relaxation process at $\sigma_1 = \sigma_{i_1}$ during the time between $\tau_0$ and $\tau_0 + d\tau_0$ will be replaced by a process with a relaxation time between $\tau = \tau_0 / f(\sigma_{i_1})$ and $(\tau + d\tau) = (\tau_0 + d\tau_0) / f(\sigma_{i_1})$, where $f(\sigma_{i_1})$ is the coefficient of the stress-time reduction, establishing an equivalent bond between the intensity of the stress and the time. The equilibrium pliability, corresponding to a microelement with a relaxation time between $\tau_0$ and $\tau_0 + d\tau_0$, is invariant with respect to changes in $\sigma_1$, i.e., $L_{\sigma_1}\Delta(\tau) = f(\sigma_0) L_{\sigma_0}[f(\sigma_0) \tau] d\tau = L_{\sigma_0}(\tau) d\tau = \text{const}.$

Consequently, in the stress-time analogy, there is postulated the experimentally verified fact [4] that, with nonlinear viscoelasticity, an increase in the stresses leads to acceleration of the relaxation processes and to a shift of the relaxation spectrum into the region of lower values of the time; under these circumstances, with a change in $\sigma_1$, the elements of area and the whole area of the relaxation spectrum remain constant, while the density of the spectrum is transformed in accordance with law of the hyperbolic rotation of the plane $L - \tau$. Then, the nonlinear rheological relationship can be written in the following form:

$$I_{ij}(t, T_0, \sigma_i) = b_{\sigma_1} \int_{-\infty}^{+\infty} L_{ij}(\ln \tau, T_0, \sigma_i) \left(1 - e^{-\frac{\tau}{\tau_0}}\right) d\ln \tau,$$

Here, the coefficients $b_{\sigma_1}$ and $a_{\sigma_1}$ reflect, respectively, the vertical and horizontal shifts of the pliability curves (with a change in $\sigma_1$) with respect to the "basic" curve $I_{ik}\Delta(t, T_0, \sigma_{i_0})$.

HYPOTHESIS III (Vibro-Time Analogy). In a majority of cases, the effect of vibrations manifests itself in an acceleration of the relaxation processes. Effects connected with significant structural changes (the accumulation of defects, hardening of the material), are observed only at relatively large amplitudes of the dynamic component [5, 7]. We limit ourselves to the case when the predominant effect of small vibra-