DESIGN OF MOMENTLESS AXIALLY SYMMETRICAL VESSELS FROM REINFORCED HEREDITARY-ELASTIC MATERIAL

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The conditions for the existence of a momentless stress distribution in a vessel made from reinforced, hereditary-elastic material and profiled as a simple or composite envelope of revolution are formulated. The possibility of realizing these conditions and controlling the creep process in a vessel by selecting a specific law governing the reinforcement of the material is established.

The danger of stress concentrations in structures made from reinforced plastics is well known. Since these materials exhibit viscoelastic properties, even when the stress concentration arising from the load is permissible, this can be under continued load since "weakening" of the material and redistribution of the stresses in the structure occurs with the passage of time.

However, the ability to control the internal structure and consequently the properties of such materials opens up wide possibilities for fighting against such undesirable effects. It is shown below that these possibilities can be utilized for the complete removal of compressive bending stresses in a vessel made from reinforced, hereditary-elastic material, and having the profile of a simple or composite envelope of rotation.

1. Relations for the Hereditary Elastic Connection between the Stresses and Strains in a Reinforced Layer

The model proposed in [1] is used as a structural model for a reinforced material. According to this model the longitudinal, normal stresses $\sigma_\alpha$ ($\alpha = 1, 2$ are the directions of the coordinate lines) in a reinforced layer can be expressed in terms of the corresponding stresses $\sigma_\alpha^{(0)}$ in the bond and the stresses in the reinforcement $\sigma^{(i)}$ ($i = 1, 2, 3, ...$ are the number of the reinforcing family of fibers) as follows:

$$\sigma_\alpha = \omega \sigma_\alpha^{(0)} + \sum_{i=1}^{l} \omega_i l_i \sigma^{(i)}$$

(we do not present expressions for the longitudinal, tangential stresses, since they will not be required in the rest of this discussion). Here, $\omega = n_i F_i / h_i$; $l_i = \cos \varphi_i$; $h_i = \sin \varphi_i$; $n_i$ is the number of fibers in the $i$-th family, required per unit length of a segment perpendicular to their direction; $F_i$ is the area of the thread cross section of the fibers in this family; $\varphi_i$ is the angle between the tangent to the trajectory of the thread and the direction $1$; $h$ is the thickness of the layer. The factor $\omega$ will be defined from one of the equations ($h_i$ is the thickness of the layer of fibers in the $i$-th family) $-\omega = 1$ or $\omega = 1 - \frac{1}{h} \sum_{i=1}^{l} h_i$.

The first equation is obtained by identifying the volume of the bond with the bulk of the whole reinforced layer (the strength of the bond is overestimated), and in deriving the second equation, the bond at the extreme layers of the reinforcement was not allowed for (strength of the bond underestimated). The presence of two equations for the factor $\omega$ enables high and low estimates of the strength properties of the...
reinforced layer describing the given structural model to be obtained. The difference between them will be the smaller the weaker the bond compared with the reinforcement.

We suggest that the bond is an isotropic, hereditary-elastic material, the stresses \( \sigma_{\alpha}^{(0)} \) and strains \( \varepsilon_{\alpha} \) of which, in the case of a planar stress distribution, are connected by the equation

\[
\sigma_{\alpha}^{(0)} = \frac{\tilde{E}}{1-\nu^2} (\varepsilon_{1} + \nu \varepsilon_{2}) ; \quad \sigma_{2}^{(0)} = \frac{\tilde{E}}{1-\nu^2} (\varepsilon_{2} + \nu \varepsilon_{1}) ,
\]

(1.2)

where \( \tilde{E} = E(1-\Gamma^*) ; \nu = \nu(1+N^*) ; \) \( E \) is the elastic modulus; \( \nu \) is the coefficient of transverse elastic expansion of the bond; \( \Gamma^* \) and \( N^* \) are operators belonging to the class of \( \Gamma \)-operators [2], while for an arbitrary function \( y \) it is assumed that \( \Gamma^* y = \int_0^1 \Gamma(t-\tau) y(\tau) d\tau \), and the operator \( N^* \), on the assumption of the absence of a bulk aftereffect, is defined in terms of \( \Gamma^* \) from the equation [2, p. 135] \( 2\nu N^* = (1-2\nu) \Gamma^* \).

We also suggest that the reinforcing elements are elastic and unidimensional, so that

\[
\sigma(t) = E \varepsilon(t) ; \quad \varepsilon^{(t)} = \varepsilon_{1} \varepsilon_{1}^{(t)} + \varepsilon_{2} \varepsilon_{2}^{(t)} .
\]

(1.3)

Drawing on relations (1.2) and (1.3) and making the necessary algebraic operations on the operators \( \Gamma^* \) and \( N^* \), we represent (1.1) in the form

\[
\sigma_{\alpha} = a_{\alpha 1} \varepsilon_{1} + a_{\alpha 2} \varepsilon_{2} ;
\]

(1.4)

\[
\tilde{a}_{\alpha \alpha} = \frac{\omega E}{1-\nu^2} (a_{\alpha \alpha} - K^*) ; \quad \tilde{a}_{\alpha 2} = 1 + \frac{1-\nu^2}{\omega E} \sum_{i=1}^{j} \omega_{i} E_{i} l_{i} \varepsilon_{i} ;
\]

(1.5)

\[
\tilde{a}_{22} = \frac{\omega E}{1-\nu^2} (a_{i 2} - M^*) ; \quad a_{i 2} = \nu + \frac{1-\nu^2}{\omega E} \sum_{i=1}^{j} \omega_{i} E_{i} l_{i} \varepsilon_{i} ;
\]

\[
K^* = (\Gamma^* + R^*_2 - \Gamma^* R^*_2) (1+R^*_1) - R^*_1 ; \quad M^* = \nu (K^* - N^* + K^* N^*) ;
\]

\[
R^*_1 = \sum_{j=1}^{j} \left( \frac{1-2\nu}{2-2\nu} \varepsilon_{j} \right) ; \quad R^*_2 = \sum_{j=1}^{j} \left( \frac{1-2\nu}{2+2\nu} \varepsilon_{j} \right) .
\]

(1.6)

The coefficients \( a_{\alpha \beta} \) must depend on the coordinates, or each of the quantities \( \omega, \omega_{i} \), \( E, E_{i}, I_{\alpha i} \) must be a function of the coordinates. We now express the center of relaxation of the bond \( \Gamma(t-\tau) \) by the expression (\( \gamma \) and \( \beta \) are constants)

\[
\Gamma(t-\tau) = \frac{\gamma}{\beta} \exp \left( -\frac{t-\tau}{\beta} \right) .
\]

(1.6)

We then have

\[
\Gamma^* y = y_{1} \Gamma^* y_{1} + y_{2} \Gamma^* y_{2} ; \quad M^* y = y_{1} \Gamma^* y_{1} - y_{2} \Gamma^* y_{2} ;
\]

\[
\Gamma^* y_{1} = \int_0^1 y(\tau) \exp \left( -\frac{t-\tau}{\beta} \right) d\tau ; \quad \gamma_{1} = \frac{1+\nu}{4} \left( 1-2\nu \right) ; \quad \gamma_{2} = \frac{3}{4} \left( 1-\nu \right) ;
\]

(1.7)

\[
\lambda_{1} = 1 - \frac{1-2\nu}{2-2\nu} ; \quad \lambda_{2} = 1 + \frac{1-2\nu}{2+2\nu} .
\]

From (1.4) we obtain the following equation for the connection:

\[
\frac{1-\nu^2}{\omega E} \sigma_{1} = (a_{11} - y_{1} \Gamma^* y_{1} - y_{2} \Gamma^* y_{2}) e_{1} + (a_{12} - y_{1} \Gamma^* y_{1} + y_{2} \Gamma^* y_{2}) e_{2} ;
\]

\[
\frac{1-\nu^2}{\omega E} \sigma_{2} = (a_{i 2} - y_{1} \Gamma^* y_{1} + y_{2} \Gamma^* y_{2}) e_{1} + (a_{22} - y_{1} \Gamma^* y_{1} - y_{2} \Gamma^* y_{2}) e_{2} .
\]

(1.8)

These equations establish the law governing the relaxation of normal stresses in a reinforced layer for a binder relaxation center given in the form of (1.6). All the coefficients and operators on the right hand side are defined from Fqs. (1.5) and (1.7).

The reverse, with respect to (1.8), creep equations are obtained by means of the Laplace transformation, allowing for the fact that the stresses \( \sigma_{\alpha} \) are independent of time. As a result of this transformation we find the description \( \varepsilon_{\alpha}(p) \) (\( p \) is a transformation parameter) of the function \( \varepsilon_{\alpha} \), i.e.,

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