FURTHER DEVELOPMENT OF THE MODEL OF A FILTRATION FLOW WITHIN THE CONCEPT OF THE EFFECTIVE DIAMETER

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Extension of the concept of the effective diameter to steady-state filtration flows allows us to suggest a generalized ideal model of a porous medium combining all the existing models (models of nonintersecting cylindrical capillaries, of parallel plane pores, etc.) and possible calculation procedures based on the notions of a filtering medium in the form of longitudinal parallel channels with a certain shape of the cross section. Results of a study of a filtration flow are presented for a porous medium. The study was carried out using the calculation model suggested.

Determination of the profit is achieved upon extension of the concept of the effective diameter [1] to isothermal steady-state filtration flows of a viscous Newtonian liquid. Whereas, in the case of using the hydraulic diameter $d_h$ as a characteristic dimension, the equations of the Darcy coefficient $\lambda_{dh} = A/Re_{dh}$ are a family of congruent straight lines on the plane $\lambda_{dh} - Re_{dh}$ in the logarithmic anamorphosis (Fig. 1a), in the case of using the effective diameter as a geometric scale

$$d_e = (A_d/A)^{0.5} d_h = K_1 d_h$$

(1)

the relations $\lambda_{de} = f(Re_{de})$ (where $\lambda_{de} = K_1 \lambda d_h$ and $Re_{de} = K_1 Re_{dh}$ are the modified Darcy coefficient and the modified Reynolds number) are described by the single straight line $\lambda_{de} = A_d/Re_{de}$ on the plane $\log \lambda_{de} - \log Re_{de}$ (Fig. 1b). This means that in a laminar flow use of $d_e$ in the definition of the modified Reynolds number and simplex $\overline{I} = l/d_e$ (the relative length) ensures strict similarity of the flows in open channels or noncircular pressure ducts to the flow in a circular pressure tube in the sense that it allows absolutely accurate hydraulic calculations of pressure and gravity flows from the formulas for a circular pressure tube with $d_e$ substituted for $d$ in them:

$$\Delta E = A_d l^2 / (2d_e Re_{de})$$

(2)

$$Re_{de} = \nu d_e / \nu \leq Re_{d}^{1.0} = 2320.$$  

(3)

When using formulas (2) and (3) for experimental investigation of laminar filtration, the parameter $d_e$ can be considered as an effective diameter of pores or a translating element in the porous medium, and the calculation model itself can be reasonably called a generalized ideal model of the porous medium since the shape of the cross-section of parallel prismatic capillaries is not specified and can be arbitrary (for example, with the cross-section as a cylinder, a flat slot, an equilateral triangle, a square, an ellipse, a trigonal or tetragonal asterisc, i.e., translating elements that are formed in the dense packing of circular rods located at the angles of the equilateral triangle or a square, etc.).

For a laminar filtration flow from the Darcy formula (2) describing the hydraulic resistance of laminar pressure and gravity flows with $i = \Delta E/(lg)$ and $v_f = v_m$, the following relation is obtained to be used for calculating the effective pore diameter:

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Fig. 1. Shapes of the equations of the Darcy coefficient \( \lambda_L \) in noncircular ducts and channels in the zone of laminar flow when hydraulic (a) and effective (b) diameters are used as a characteristic dimension \( L \).

\[
d_e = \left[ \frac{A_d \nu f}{(2mg)} \right]^{0.5}.
\]

From relation (4) with \( K = \nu f/\alpha g \) the following formula can be obtained:

\[
d_e = (0.5 A_d K/m)^{0.5},
\]

which relates the effective pore diameter to the permeability coefficient of the porous medium \( K = C \nu /g \), where \( C \) is the filtration coefficient entering into the linear relation suggested by Darcy

\[
\nu = C_i.
\]

If this relation, which in the form of the Poiseuille relation for pressure and gravity flows has the form \( \nu = m \Delta \nu = mAE2d_e^2/(h) \), were observed strictly, then without clogging of the pores, every porous medium could be characterized in terms of constant values of \( d_e \) or \( K \). As can be seen from numerous experimental data [2], the hypothetic formula (6) should be recognized as invalid. In laminar filtration, because of the complexity of the filtration process itself (unsteady flow, differences in individual characteristics of the porous media, presence of numerous concurrent hydrodynamic phenomena [2], etc.), the coefficients or parameters that characterize integral properties of the porous medium cannot and must not remain constant in the range \( Re_d = 0 \rightarrow 2320 \). All the aforementioned statements are confirmed indirectly by the following parameters. In the case of stabilized curvilinear liquid flow that is more complicated in comparison with stabilized flows in tubes and channels and less simple in comparison with filtration at a fixed curvature parameter (the ratio of the diameter of the coil to the diameter of the tube), with increase in the Reynolds number \( Re_d \), the following resistance zones are changed [3]: the zone of laminar flow, where curvilinearity of the flow has no effect on the hydraulic resistance; the zone of laminar flow with increasing macrovortices, where the resistance is affected not only by the Reynolds number but also by the curvature parameter; the zone of transient flow with macrovortices; the zone of a turbulent flow with increasing damping of macrovortices, where the hydraulic resistance is affected by the Reynolds number and curvature; the zone of turbulent flow, where the curvature no longer affects the resistance. In nonstabilized flow in valve devices that is simpler than a filtration flow, five zones of hydraulic resistance are distinguished in [4], depending on the character of the effect of the Reynolds number \( Re_d \) on the coefficient \( \xi \); in the first zone \( \xi = B/Re_d \), where the flow is laminar in the pipeline and in the local resistance; in the second zone \( \xi = D/Re_d^2 \), where the laminar flow is violated in the local resistance; in the third zone \( \xi = 8Re_d^{0.53} \), where the laminar flow is violated in the pipeline too; in the fourth and fifth zones \( \xi = F \), where the Reynolds number is low or has no effect on the coefficient \( \xi \) at all.