EVAPORATION OF LIQUIDS FROM CYLINDRICAL VESSELS UNDER CONDITIONS OF FREE CONCENTRATIONAL CONVECTION IN A GAS PHASE

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An analytical solution is obtained for the axisymmetric problem of free concentrational convection in a vapor-gas mixture with isothermal evaporation of liquids from open cylindrical vessels. Formulas are derived to calculate concentration fields, local and integral mass fluxes of vapor. A comparative analysis of the results of analytical and numerical simulation is carried out for the processes of the evaporation of liquids under the conditions of convective mass transfer.

During evaporation of liquids whose molar mass is smaller than that of the gas in whose medium the evaporation occurs, conditions can develop under which the hydrodynamic stability of the gas phase is disturbed. In this case, mass transfer of the vapor takes place in the free convection regime and is described by the system of equations [1]:

\[- \nabla p_m + \eta \Delta V - \rho g = 0,\]

\[\nu \cdot \nabla \rho_1 - D \Delta \rho_1 = 0,\]

\[\text{div} \nu = 0,\]

where \(p_m, \rho, \nu\) are the pressure, density, and velocity of the gas-vapor mixture; \(\eta, D\) are the coefficients of the dynamic viscosity and diffusion; \(\rho_1 = m_1/V\) is the concentration of the vapor in the mixture.

The steady-state evaporation regime in cylindrical vessels can be analyzed in the approximation \(v_r \ll v_z\) (where \(v_r\) and \(v_z\) are the radial and axial velocity components), which is the more correct the smaller the radius of the vessel \(R\) in comparison with its height \(H\). In this case we do not take into consideration the processes occurring in the adjoining layer of the vapor-gas mixture, whose thickness is negligibly small as compared with the dimensions of the vessel. For an axisymmetric variant of convection, which was studied by numerical methods in [2], in this approximation we can obtain an analytical solution to the system of Eqs. (1)-(3) which in cylindrical coordinates takes the form:

\[- \frac{\partial p}{\partial z} + \eta \left( \frac{2 \partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} \right) - \rho g = 0,\]

\[v_z \frac{\partial \rho_1}{\partial z} - D \left( \frac{2 \rho_1}{\partial r^2} + \frac{1}{r} \frac{\partial \rho_1}{\partial r} + \frac{\partial^2 \rho_1}{\partial z^2} \right) = 0.\]

The pressure and density of the mixture will be written as

\[p_m = -\rho_0 g z + p, \quad \rho = \rho_0 + \rho',\]

where \( \rho_0 \) is the density of the mixture in the medium surrounding the vessel; \( \rho, \rho' \) are the excess values of the pressure and density caused by the concentrational inhomogeneity of the mixture.

Assuming the change in the pressure to be much smaller than that in the molar mass of the mixture \( M \), we write \( \rho' = \rho (1 - \rho_0/\rho) = (1 - M_0/M) \), just as in [2]. Using the expression for the molar mass of the mixture \( M = M_1 M_2 / (\omega_1 M_2 + \omega_2 M_1) \), where \( \omega_1 = \rho_1/\rho, \omega_2 = \rho_2/\rho, M_1 \) and \( M_2 \) are the mass fractions and the molar masses of the components, we find

\[
\rho' = \rho \frac{(M_2 - M_1)(\omega_{01} - \omega_1)}{M_1 + \omega_{01}(M_2 - M_1)} .
\] (7)

Substituting expressions (6)-(7) into Eqs. (4)-(5) and introducing the new concentrational variable \( \psi = (\omega_1 - \omega_{01})/(\omega_{1e} - \omega_{01}) \) \( (\omega_{1e} \) is the equilibrium mass fraction of the vapor), we obtain a system of equations in dimensionless cylindrical coordinates \( x = r/R \) and \( y = z/R \):

\[
-\frac{\partial F}{\partial y} + \frac{\partial^2 u}{\partial x^2} + \frac{1}{x} \frac{\partial u}{\partial x} + Ra \psi = 0 ,
\] (8)

\[
u \frac{\partial \psi}{\partial y} - \frac{\partial^2 \psi}{\partial x^2} + \frac{1}{x} \frac{\partial \psi}{\partial x} + \frac{\partial^2 \psi}{\partial y^2} = 0 ,
\] (9)

where \( F \) and \( u \) are the dimensionless pressure and velocity; \( Ra \) is the Rayleigh number defined by the expressions

\[
F = \frac{b R^2}{\rho v D} \quad \text{and} \quad Ra = Pr Gr ; \quad Pr = \nu/D ; \quad Gr = \frac{\alpha g R^3}{\nu^2} ,
\] (10)

here \( \alpha = (M_2 - M_1)(\omega_{1e} - \omega_{01})/ [M_1 + \omega_{01}(M_2 - M_1)] \), \( Pr \) and \( Gr \) are the Prandtl and Grashof numbers.

Now we will seek the function \( \psi \) in the form:

\[
\psi (x, y) = \varphi (x) - Ay + B ,
\] (11)

which allows us to transform system of Eqs. (8)-(9) in the following manner:

\[
\frac{\partial^2 u}{\partial x^2} + \frac{1}{x} \frac{\partial u}{\partial x} + Ra \varphi = 0 ,
\] (12)

\[
-\frac{\partial F}{\partial y} + Ra (Ay - B) = 0 ,
\] (13)

\[
-Au = \frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{x} \frac{\partial \varphi}{\partial x} .
\] (14)

Applying the \( \Delta_x \) operation to Eq. (12) and using Eq. (14), we obtain a biharmonic equation for the velocity \( u \):

\[
\Delta^2 u - Ra Au = 0 .
\] (15)

The solution satisfying the requirement for the existence of an adherent layer at the side surface of the vessel and of the continuity equation, has the form:

\[
u (x) = C \left[ \frac{J_0 (kx)}{J_0 (k)} - \frac{I_0 (kx)}{I_0 (k)} \right] , \quad k = 4 \sqrt{Ra} A .
\] (16)

After the substitution into Eq. (14), this equation makes it possible to find the function \( \varphi (x) \):

\[
\varphi (x) = \frac{AC}{k^2} \left[ \frac{J_0 (kx)}{J_0 (k)} + \frac{I_0 (kx)}{I_0 (k)} \right] .
\] (17)