The local overheating at the tips of growing cracks in cyclically loaded plastics is examined on the assumption that there is almost no change in overheating up to failure (relatively small amplitudes). A method is proposed for estimating the fatigue life of solids and polymers with allowance for local overheating or local aging of the material. A comparison of the formulas obtained with the experimental data confirms the importance of overheating in cyclic loading as compared with other manifestations of the local relaxation processes.

In [1, 2] it was shown that the application of Bailey's criterion to the cyclic loading of plexiglas gives agreement between the calculated and experimentally measured lives \( t_{CY} = t_p \) only for a small number of cycles to failure. Similar results for a number of solids were then obtained by other authors [4–6, etc.].

Various explanations of these findings have been proposed: local overheating at the tips of cracks [1, 2, 6], stress relaxation [4], and even mechanochemical effects [5]. It has also been observed that the reason may be the inaccuracy of criterion (1).

The latter, as shown in [7, 8], leads to a certain exaggeration of the fatigue life: the direct calculation of \( t_{CY} \) from the crack growth rate for a large number of cycles to failure gives values of \( t_{CY} \) approximately 10% smaller than the \( t_{CY} \) calculated from Eq. (1). This small difference cannot explain the sharp difference found between the calculated and experimental values of \( t_{CY} \) (Fig. 1).

The discrepancy between experiment and calculation is evidently attributable to various relaxation effects associated both with stress relaxation and structural changes (which affect the structure-sensitive coefficient \( \gamma \) in the equation for the time dependence of the strength) and with overheating caused by the work of cyclic deformation. These effects should be especially marked at the points of stress concentration—the crack tips.

The relative role played by stress relaxation and overheating may vary with the test conditions. These two factors cannot be differentiated on the basis of the experimental data: a decrease in \( t_{CY} \) as compared with the calculated value indicates a decrease in the coefficient \( \alpha = \gamma/kT \), but this decrease may be caused either by a decrease in \( \gamma \) or by an increase in the temperature \( T \) by a certain amount \( \Delta T \), or, most frequently, by both these effects. However, there are two limiting cases in which only one of the effects is of practical importance. In the presence of cyclic loads with a period on the order of 24 hr the discrepancy between experiment and theory can only be associated with a change in the effective (time-averaged) value of \( \gamma \) [9]. On the other hand, at a high cyclic loading frequency the overheating, accelerating both the failure process and stress relaxation, should be considerable. Stress relaxation is fairly rapid and ceases to play an important part in the deviation of the experimental from the theoretical curves.

Let us consider the second case corresponding to a fairly high cyclic loading frequency, when the discrepancy between theory and experiment is determined exclusively by overheating. For convenience, we consider only saw-tooth cycles (Fig. 2), which, however, does not restrict the generality of the argument. Moreover, we confine ourselves to temperatures remote from the glass-transition temperature, i.e., we exclude from consideration the nonstationary self-heating process due to the increase in mechanical loss factor with increase in temperature [10].

The front of a growing crack is a linear heat source; its velocity and the temperature field near the front are related in a complicated way. The nature of this dependence can be established starting from the ideas of the fluctuation theory of strength [7, 11]. However, for engineering fatigue life formulas for plastics with allowance for local overheating it is more convenient to consider a simplified variant of the problem.

The energy released in each loading cycle at the tip of a crack per centimeter of the length of the crack front is equal, in first approximation, to the work of cyclic deformation \( \delta A \) multiplied by the mechanical loss factor \( \kappa \):

\[
\delta Q = \kappa \delta A = \frac{\beta \sigma^2 (1 - K^2)}{2E \left(1 - \frac{L}{L}^2 \right)^2}.
\]
where $\beta$ is the stress concentration coefficient at the crack tip; $l$ is the crack length; $L$ is the width of the strap specimen; $3\sigma/(1 - l/L)$ is the true stress at the crack tip; $d$ is the linear dimensions of the overstress region at the crack tip, in which most of the heat is generated; $\varepsilon$ is Young’s modulus; $K = \sigma_\varepsilon/\sigma_2$ is the cycle asymmetry coefficient (see Fig. 2).

We assume that the energy $\delta Q$ increases the temperature of a certain volume $\Delta V$ near the crack tip by an amount $\Delta T$. In reality, there is a temperature gradient near the moving tip and $\Delta V$ and $\Delta T$ are certain average characteristics. The temperature rise $\Delta T$ is equal to

$$\Delta T = \frac{\delta Q}{\rho \Delta V} = \frac{\kappa^2 \sigma_2^2 (1-K) \delta^2}{2 E \rho \left(1 - \frac{l}{L}\right) \Delta V},$$

where $\rho$ is the specific heat of the material (per cubic centimeter).

If the length of the crack grows during a cycle of duration $\tau$ by an amount $\Delta l$, then, in first approximation, we can set $\Delta V = a \Delta l$, where $a =$ const signifies the thickness of the "heated" layer in a direction perpendicular to the crack front. The alternative assumption $a \sim \Delta l$ (rate of heat propagation proportional to temperature difference) leads to practically the same results. For a saw-tooth cycle and large number of cycles to failure the cyclic mean crack growth rate $\bar{\nu}$ is equal to [8].

$$\bar{\nu} = \frac{1 - l}{L} \alpha \sigma_2 (1-K) \nu_2,$$

where $\nu_2$ is the rate of growth of a crack of length $l$ in a strip specimen of width $L$ at constant stress $\sigma_2$; $\alpha = \gamma/k (1/T - 1/T_p)$ is the constant in the equation of the time dependence of the strength; $T_p$ is the pole temperature [12]. Substituting expression (4) into (8) and keeping in mind that the crack growth rate corresponds to the local temperature $T + \Delta T$, we find

$$\Delta T = \frac{\kappa^2 \sigma_2^2 (1-K)^2 (1+K) \left(1 - \frac{T + \Delta T}{T_p}\right) \beta \delta^2}{2 E \rho \alpha \sigma_2 (1-K) \nu_2},$$

where $\nu_2$ is the crack growth rate at $\sigma = \sigma_2$ and the temperature $T + \Delta T$.

As may be seen from Eq. (5), the local temperature rise does not remain the same during the failure process. At first, at $l \ll L$

$$\Delta T = \frac{\kappa^2 \sigma_2^2 (1-K)^2 (1+K) \left(1 - \frac{T + \Delta T}{T_p}\right) \beta \delta^2}{2 E \rho \alpha \sigma_2 (1-K) \nu_2},$$

where $\nu_2$ is the initial crack growth rate at $\sigma = \sigma_2$ and a temperature $T + \Delta T$. As $l$ increases, $\Delta T$ varies from $\Delta T_0$ to a limiting value $\Delta T_{cr}$ corresponding to the critical crack growth rate $\nu_{cr}$ at the actual stress in the cross section of the cracked specimen $\sigma_{cr} = \sigma_2/l - l_{cr}/L$:

$$\Delta T_{cr} = \frac{\kappa^2 \sigma_2^2 (1-K)^2 (1+K) \left(1 - \frac{T + \Delta T_{cr}}{T_p}\right) \beta \delta^2}{2 E \rho \alpha \sigma_2 (1-K) \nu_{cr}}.$$

Since the principal contribution to the fatigue life is made by the initial stage of failure, the deviation of the experimental from the theoretical fatigue life data is primarily associated with the initial local overheating $\Delta T_0$.

In accordance with the theoretical formula [1], the fatigue life $t_{cy}$ for saw-tooth cycles is given by

$$t_{cy} = \frac{\alpha \sigma_2 (1-K)}{1 - e^{-\alpha \sigma_2 (1-K)}} \tau_2,$$

where $\tau_2 = A e^{-\alpha \sigma_2}$ is the life at constant stress $\sigma_2$; $A$ and $\alpha$ are coefficients in the equation for the time dependence of the strength. In accordance with the fluctuation theory of strength [7,11],

$$\tau_2 = \frac{L}{\alpha \sigma_2 \nu_2},$$

where $\nu_2$ is the initial crack growth rate at $\sigma = \sigma_2$.

Thus, $t_{cy} \sim 1/\nu_2$, which confirms the importance of the initial overheating $\Delta T_0$ for $t_{cy}$.

We now propose a semiempirical method of calculating the fatigue life of plastics based on the substitution of the quantity $T + \Delta T_0$ for $T$ in Eqs. (8) for $t_{cy}$. In accordance with (6), the local initial overheating itself depends both on the characteristics of the material ($E, \rho, \ldots$), the shape and size of the specimen, and the cyclic loading conditions ($\sigma_2, K, \theta, T$).

As the first approximation, we set $\Delta T_0 + T_0 \approx T$ in the right-hand side of Eq. (6). Then

$$\Delta T_0 = \frac{\kappa^2 \sigma_2^2 (1-K)^2 (1+K) \beta \delta^2}{2 E \rho \alpha \sigma_2 \nu_2 (T)},$$

and we write $\Delta T_0$ in the form of a product of two functions:

$$\Delta T_0 = C \Phi_\varepsilon,$$

where $C$ is a function of the material and geometry of the specimens and $\Phi_\varepsilon = \sigma_2^2 (1-K)^2 (1+K) \tau_0$ is the cyclic loading characteristic. The constant $C$ for a given batch of specimens (parts) should be determined experimentally for one or two specific loading cycles from the value of $\Delta T_0$ corresponding to the deviation of...