Hence

\[ D^x(t) = \left[ 1 - \frac{G_H - G_e}{G_H} e^{-\frac{\sigma}{\tau^v}} \right] \frac{G_e}{G_H} \frac{G}{\tau^v} \]  

Substituting (4) and (6) in Eq. (3), we obtain an expression for the Poisson's ratio \( \mu(t) \):

\[ \mu(t) = \frac{\frac{\sigma}{\tau^v} \left( \frac{G_H - G_e}{G_H} e^{-\frac{\sigma}{\tau^v}} \right) \left( \frac{G_H - G_e}{G_H} e^{-\frac{\sigma}{\tau^v}} \right)}{\frac{G_H - G_e}{G_H} e^{-\frac{\sigma}{\tau^v}} \left( \frac{G_H - G_e}{G_H} e^{-\frac{\sigma}{\tau^v}} \right)} \frac{G_H}{G_H} \frac{G}{\tau^v} \]  

It is clear from Eq. (7) that in creep the deformation properties of polyethylene are determined by two elastic coefficients of distortion–G_H and G_E, an elastic coefficient of change of volume K_H, and two strain relaxation–time coefficient (shape v_s and volume v_r). For the unique determination of the coefficients we used the experimental results (see Figs. 2 and 3).

It was found that the volume strain-relaxation time \( v_r = 16 \) days, the shear strain-relaxation time \( v_s = 6 \) days, the coefficient \( G_H = 475 \) kg/cm^2, and the coefficient \( G_E = 174 \) kg/cm^2; the modulus \( K_H = 1010 \) kg/cm^2.

Substituting the characteristics of the material in (7), we obtain an expression for its Poisson's ratio:

\[ \mu(t) = \frac{1 - 0.75e^{-0.04t}}{2 - 1.15e^{-0.04t}} \]  

The \( \mu(t) \) curve obtained from (8) is shown in Fig. 1. The good agreement between the theoretical \( \mu(t) \) curve and the experimental data indicates that our theory concerning the process of polyethylene creep under tensile load is a reliable one.

REFERENCES

1. I. V. Shamov, Mekh. polim. [Polymer Mechanics], 8, 52, 1965.

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is an exponential expression that can be transformed into the equation
frequently employed for the left-hand part of the long-term-strength curve of GRP: $f_b = A e^{-\alpha_0 b}$.

Using Eqs. (2) and (3), we obtain
$$f = e^{(\theta - \alpha_0 b)(D_f - D_i)}.$$  \hspace{1cm} (4)

Solving Eqs. (3) and (4) together, after a number of transformations, we obtain an expression for the variable damage in the form
$$D = D_f + \frac{\ln \frac{f}{f_b}}{\ln \frac{f}{f_b} - \ln \frac{f}{f_b}} (D_f - D_i).$$ \hspace{1cm} (5)

If we assume that $f_1$ is equal to unit time, then
$$D = D_f + \frac{\ln \frac{f}{f_b}}{\ln \frac{f}{f_b} - \ln \frac{f}{f_b}} (D_f - D_i).$$ \hspace{1cm} (6)

The quantity $D_f$ is found from the expression
$$D_f = 1 - \Phi(\frac{\theta_i - \alpha_0}{\bar{s}}) + \Phi(\bar{s})$$ \hspace{1cm} (7)

where $\theta_i$ is the ultimate strength, the experimental value for the given area; $\theta_0$ is the initial stress; $\Phi$ is the remaining finite destroyed area before final failure in short-term tests. The quantity $D_f$ is determined starting from the given stress level and the law of distribution of strength over area elements of the cross-section of the GRP specimen or part [1] and

$$D_f = 1 - \Phi(\frac{\theta_i - \alpha_0}{\bar{s}}) + \Phi(\bar{s}),$$ \hspace{1cm} (7)

where $\bar{s}$ is the average strength of the area elements composing the cross-section; $\bar{s}$ is the standard deviation of the strength of these elements; and $\Phi$ is the Laplace function. Then the quantity $\theta_0$ is found from the equation
$$\theta_0 = (1 - \nu A)\{\Phi(\theta_0) + \Phi(\bar{s})\},$$ \hspace{1cm} (8)

where $(1 - \nu A)[\Phi(\theta_0) + \Phi(\bar{s})]$ must be maximum; and $\nu$ is the coefficient of variation of the strength of the area elements over the cross-section.

We investigated elements of a GFRP based on phenol-formaldehyde resin (AG-4V and AG-4S). As the experiments showed, the long-term-strength curve is well described by an exponential relation (Fig. 1). The curve was constructed on the basis of average values of the specimen life at the given stress level, 10 specimens being tested at each level.

For the GFRP in question $\theta_0 = 30.2$ kg/mm$^2$ with $\nu = 0.16$; then $\lambda_f = 1.25$, and we obtain $1 - \nu A[\Phi(\theta_0) + \Phi(\bar{s})]$, whence $\bar{s} = 42.1$ kg/mm$^2$ and $\bar{s} = 0.075$ kg/mm$^2$. For short-time loading the relative area of damage before final failure is 0.014 or 0.14% of the entire area.

Nonstationary long-term static loading was carried out according to the program represented in Fig. 2. At the given stress levels damage accumulated with time as shown in the same figure. It is clear that for the higher initial stress the initial damage is greater and the final damage less as compared with the same characteristics for the lower stress level, since the damage accumulation curves in $D - r/r_b$ coordinates intersect. Starting from the given program we calculated theoretically that the number of units (a time unit includes two succes-

<table>
<thead>
<tr>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$D_1$</th>
<th>$D_2$</th>
</tr>
</thead>
<tbody>
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<td>21 kg/mm$^2$</td>
<td>23 kg/mm$^2$</td>
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<td>0.0025</td>
</tr>
<tr>
<td>0.373</td>
<td>0.313</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2

The number of units may be calculated graphically as follows. From the given time of operation of the stress $\sigma_1$ per unit $\tau'$ we find the ratio $\tau'/\tau_i$. Then the point $C'$ corresponds to damage $D'$ at $\sigma_1$ in time $\tau'$. Since according to the program after time $\tau'$ at stress $\sigma_1$ the stress $\sigma_2$ acts for a time $\tau''$, we can determine the ratio $\tau''/\tau_f$ (where $\tau_f$ and $\tau_f'$ are the times to failure corresponding to stress $\sigma_2$ and $\sigma_1$, respectively under long-term static loading, for the total number of units, the equation
$$n = \frac{1}{\tau'} + \frac{1}{\tau''} - 1,$$ \hspace{1cm} (9)

gives a value equal to 10 units. The actual number of units survived by 10 specimens is shown in Table 1.

To calculate the theoretical value of the number of units it is necessary to know $D_1$ and $D_2$, which are given in Table 2 for the given stress levels.

The number of units may be calculated graphically as follows. From the given time of operation of the stress $\sigma_1$ per unit $\tau'$ we find the ratio $\tau'/\tau_i$. Then the point $C'$ corresponds to damage $D'$ at $\sigma_1$ in time $\tau'$. Since according to the program after time $\tau'$ at stress $\sigma_1$ the stress $\sigma_2$ acts for a time $\tau''$, we can determine the ratio $\tau''/\tau_f$ (where $\tau_f$ and $\tau_f'$ are the times to failure corresponding to stress $\sigma_2$ and $\sigma_1$, respectively). To the point $C'$ on the damage curve for $\sigma_1$ there corresponds a point $C''$ on the damage curve for $\sigma_2$; while to the damage $D'$ on the curve for $\sigma_1$ there corresponds at a time $\tau''$ on the damage curve for $\sigma_2$. Now to the relative time $\tau'/\tau_f$ we add the relative time $\tau''/\tau_f$, and from the damage curve we determine the damage $D''$ corresponding to the point $E''$. The damage $D''$ corresponds to the point $E'$ on the damage curve for $\sigma_1$. This computation process continues until it becomes impossible to pass from the $\sigma_1$ curve to the $\sigma_2$ curve. This corresponds to the instant of failure. After this the critical number of units or the number of units to failure is calculated.

We conclude that overloads, even if they act only for a relatively short time, may considerably reduce the total life of a part or the structure as a whole. Accordingly it is necessary to eliminate overloads as far as possible or take them into account in calculating service life on the basis of damage accumulation.