ALLOWANCE FOR GAS BLOWING
IN SUPERSONIC FLOW OVER A WEDGE

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The static pressure on the dividing streamline in intense blowing of a gas through the wall of a wedge in a supersonic flow is determined.

In several investigations ([1, 2], and others) the experimental data were presented basically in the form of angle of inclination of the contact surface. Bott [1] discussed the effect of the viscosity on the dividing streamline on the experimental results, but presented no numerical results for the influence of this factor on the shape of the dividing streamline.

The present author attempts to show how to determine the numerical effect of viscosity during intense blowing on a wedge on the position of the contact surface, using the relations at the shock wave and a correlation of [1].

We consider the exact relation of [3] for a perfect gas; this relates the flow deviation and the pressure difference in passing through a shock wave. When gas is blown through the wall of the wedge and the shock wave is attached, one can write this relation in the form

\[
\tan (\theta + \Delta \theta) = \sqrt{M_\infty^2 - 1} \frac{C_p}{2} \left[ \frac{1 - \frac{\gamma + 1}{2} \frac{M_\infty^2 - 1}{C_p} - \frac{C_p}{2}}{1 - \frac{\gamma - 1}{2} M_\infty^2 \frac{C_p}{2}} \right]^{\frac{1}{2}}.
\]

Carrying out a transformation in Eq. (1), we obtain

\[
f(\phi) = C_p^3 - \frac{4}{\gamma + 1} \left[ 1 + \frac{\gamma - 1}{2} M_\infty^2 \frac{C_p}{2} + \frac{4}{\gamma + 1} \frac{1}{M_\infty^2} \right] \sin^2(\theta + \Delta \theta) + 16 \sin^2(\theta + \Delta \theta) C_p + \frac{16 \sin^2(\theta + \Delta \theta)}{(\gamma + 1) M_\infty^2} = 0,
\]

where \( \Delta \theta \), according to [1], is given by the relation

\[
\tan \Delta \theta = -B_p \left( \frac{R}{M} \frac{T_w}{\gamma} \right)^{\frac{1}{2}}.
\]

Since the pressure can be determined by calculating the viscous interaction, we can solve Eq. (2) by the method of successive approximations.

1. First Approximation \((Re_x = \infty)\)

Even in this simplest case it is difficult to solve Eq. (2) in the general form. Therefore, we use a result from thin body hypersonic theory, which gives only an approximation, but a value so close to the true root \( \bar{p} \), that we can use certain methods to obtain further improvement. Let

\[
\Delta \theta \approx \frac{\psi}{p} = \frac{m \left( \frac{R}{M} \frac{T_w}{\gamma} \right)^{\frac{1}{2}}}{Bp_m}.
\]
Then

\[ \bar{p} = 1 + \gamma M^2 \left( \theta + \frac{\Psi}{\bar{p}} \right)^2 \left[ \frac{\gamma + 1}{4} + \sqrt{\left( \frac{\gamma + 1}{4} \right)^2 + \frac{1}{M^2 \left( \theta + \frac{\Psi}{\bar{p}} \right)^2}} \right]. \]  

(3)

Since \( \bar{\rho} \geq 1 \), the quantity \( \frac{\gamma - 1}{\gamma + 1} \) can be neglected in comparison with \( \bar{\rho} \) and we obtain the following result from Eq. (3):

\[ \bar{p}^3 - \left[ 4 + \gamma (\gamma + 1) K^2 \right] \bar{p}^2 + \left[ 1 - \gamma (\gamma + 1) K_0 \bar{K}_e \right] \bar{p} - \frac{\gamma (\gamma + 1)}{2} K^2 = 0. \]  

(4)

We solve this equation by the method described in [4].

Introducing the notation

\[ x = \alpha \bar{\rho}, \quad a = -\alpha - \frac{4 + \gamma (\gamma + 1) K^2}{2}, \quad b = \alpha^2 \left[ 1 - \gamma (\gamma + 1) K_0 \bar{K}_e \right], \quad \alpha = \sqrt{\frac{2}{\gamma (\gamma + 1) K^2}} \]

into Eq. (4), we obtain

\[ f(x) = x^3 + ax^2 + bx - 1 = 0. \]  

(5)

If \( \alpha > 2 \), then

\[ \frac{1}{2} \sqrt{\gamma (\gamma + 1)} > K_0 \geq 0, \quad -\alpha K_0 + \sqrt{\alpha^2 K_0^2 + \frac{2\alpha - 4}{\gamma (\gamma + 1)}} > K_0 \geq 0 \]

(6)

and the function \( f(x) \) changes sign in the interval \([0, 1]\). Here \( f(0) < 0, f(1) > 0 \). From the last inequalities and the expression \( x_1 x_2 x_3 = 1 \) it follows that only one real root falls between 0 and 1. Using the third Chebyshev polynomial

\[ T_3(x) = 32 x^3 - 48 x^2 + 18 x - 1, \]  

(7)

constructed in the interval \([0, 1]\), we can reduce Eq. (5) to a quadratic. The root of this equation lying in the range \([0, 1]\) gives a value for the dimensionless pressure \( \bar{p} = \bar{\rho} \), less than 1. Therefore, we find the desired root \( \bar{\rho} \) from the equation

\[ \bar{p}^3 + \left( \bar{p} + \frac{a}{\alpha} \right) \bar{p} + \bar{p}_* \left( \bar{p} + \frac{a}{\alpha} \right) + \frac{b}{\alpha^2} = 0. \]

(8)

Since of the two roots of this equation one is physically nonreal (less than 1), we finally obtain

\[ \bar{\rho}_1 = \sqrt{\left( 1 + \frac{\gamma (\gamma + 1) K^2}{4} - \frac{\bar{p}_*}{2} \right)^2 + 2 \gamma (\gamma + 1) K_0 K_0 - 1 + \bar{p}_* \left( 2 + \frac{\gamma (\gamma + 1) K^2}{2} - \bar{p}_* \right) - 1 + \gamma (\gamma + 1) K^2 - \frac{\bar{p}_*}{2} \]  

where

\[ \bar{\rho}_* = \frac{(0.5625 - b) + \sqrt{(0.5625 - b)^2 + 4 \cdot 0.9687 \left( a + \frac{3}{2} \right)}}{2a \left( a + \frac{3}{2} \right)} . \]

If \( \alpha < 2 \), then

\[ K_0 > \frac{1}{2} \sqrt{\gamma (\gamma + 1)} \], \( K_0 \geq 0 \)

(9)

and the function \( f(x) \) will still be negative at the point \( x = 1 \), and so the root of Eq. (5) will be located in the range \([1, \infty]\). We map this interval using the transformation \( \bar{x} = \frac{1}{x} \) in the range \([0, 1]\). In this case Eq. (5) becomes the equation

\[ f(\bar{x}) = -\bar{x}^2 + bx + ax + 1 = 0. \]

(10)