§1. The linear problem of the deformation process was studied on sloping viscoelastic shells sustaining, before loading, given initial axial stresses [1]; materials with an exponential creep kernel were examined. Let us assume a surface over which a shell is outlined, such that the system of equations of the shell theory with zero boundary conditions and given initial conditions is reduced to a single resolvent equation with constant coefficients at zero boundary conditions and given initial conditions

\[ C(A_1F_t - \lambda B_1F_t) + (A_2F_t - \lambda B_2F_t) = Z [F = F(x, y, t)]; \]

\[ G_jF|_{r=0} = 0; \quad j = 1, 2, 3, 4; \quad F(x, y, 0) = f(x, y) \quad [Z = Z(x, y)]. \]

Here \( F \) is the resolvent function; \( x, y \) are rectangular coordinates; \( t \) is time; \( A_k, B_k, C, G_j \) are operators \((k = 1, 2)\); the operators \( A_1, B_1 \) express the instantaneous characteristics of the shell material; \( A_2, B_2 \) are the long-term characteristics; \( \lambda \) is a loading parameter; \( Z \) is a small fixed load, normal to the shell surface; \( f(x, y) \) is a function describing the initial state of the shell under a load \( Z \); and \( \Gamma \) is the contour of the shell.

It should be stressed that when dimensionless notation is used, the operators \( A_1, A_2 \) have a small parameter \( \varepsilon = (h/R)^{1/2} \) (\( h \) is the thickness of the shell and \( R \) is its characteristic linear dimension); this small parameter \( \varepsilon \) is among the high-order derivatives.

We shall assume that for linear self-adjoint problems of shell stability

\[ A_1q_{ik} - \lambda_{ik}B_1q_{ik} = 0; \quad G_jq_{ik}|_{r=0} = 0; \quad j = 1, 2, 3, 4 \quad (k = 1, 2), \]

where a definite thickness of walls for a thin shell has been selected, spectra have been calculated, whereby to ordered eigenvalues

\[ 0 < \lambda_{1k} \leq \lambda_{2k} \leq \lambda_{3k} \leq \ldots \]

there correspond orthonormalized eigenfunctions

\[ \begin{align*}
q_{ik} & = \eta_{ij}; \\
A_1q_{ik} & = \lambda_{ik}\eta_{ij} \quad (i, j = 1, 2, \ldots).
\end{align*} \]

Here \( \eta_{ij} \) are Kronecker symbols (\( \eta_{ij} = 0 \) for \( i \neq j \); \( \eta_{ij} = 1 \) for \( i = j \)); \( \varphi, f \) is the scalar product of the \( \varphi \) and \( f \) functions. We note that: 1) to the eigenfunctions \( \lambda_{1k}, \lambda_{2k} \) with the same indices \( k \), in general, there always correspond entirely different eigenfunctions, even if the systems of eigenfunctions \( \varphi_{1k}, \varphi_{2k} \) coincide up to normalization constants; 2) the spectra of certain stability problems have accumulation points as \( \varepsilon \to 0 \), and, moreover, these accumulation points may be at the beginning of the spectrum [2] (see specific examples in Sec. 4).

Suppose the loading parameter \( \lambda \) in Eq. (1.1) differs little from the first eigenvalues \( \lambda_{1k} \) [see (1.3)] and, perhaps, from several subsequent ones; more accurately,

\[ \begin{align*}
\lambda_{1k} - \lambda & = \delta_k\lambda_{1k}^{(l)}; \quad i = 1, 2, \ldots, l_k \quad (k = 1, 2); \\
\delta_k & \ll 1; \quad \lambda_{ik}^{(l)} > 0; \quad \lambda_{ik}^{(0)} = 0 (1); \quad l_k \gg 1.
\end{align*} \]
The \( l_k \) numbers are the multiplicities of the smallest eigenvalues of certain modified eigenvalue and eigen-
function problems [see (2.2)].

§2. To determine the initial state \( F(x, y, 0) \), we shall construct an asymptotic representation of the
solution of the equilibrium problem for a sloping shell at given stresses in the middle surface [3]:

\[
A_i F - \lambda B_i F = Z; \quad G_j F \mid \Gamma = 0; \quad j = 1, 2, 3, 4.
\] (2.1)

Here \( F = F(x, y, 0) \) is a function describing the initial state of the shell (for the notation see Sec. 1). To the
nonhomogeneous problem (2.1) there corresponds an eigenfunction and eigenvalue problem (1.2) at \( k = 1 \) for
which the relationships (1.3)-(1.5) hold.

We shall now proceed to construct an asymptotic representation of the solution of the initial-value prob-
lem (2.1); during the construction of the solution, we must find a relation between the two small parameters \( \delta \) and \( \epsilon \), the first of which is determined by the loading parameter \( \lambda \) [see (1.5)] and the second, by the spec-
trum of problem (1.2) at \( k = 1 \). An auxiliary (modified) eigenfunction and eigenvalue problem is considered:

\[
A_{0i} \Phi_{0i} - \Lambda_{0i} \Phi_{0i} = 0; \quad G_{0j} \Phi_{0j} \mid \Gamma = 0; \quad j = 1, 2, 3, 4.
\] (2.2)

Here the operators \( A_{0i}, B_{0i} \) coincide up to \( \delta_i \) with the operators \( A_i, B_i \):

\[
A_i = A_{0i} + \delta_i A_{1i}; \quad B_i = B_{0i} + \delta_i B_{1i}.
\] (2.3)

Remark. For the long-term characteristics of the material, the formally modified eigenfunetion and
eigenvalue problem coincides with problem (2.2), if we replace in the latter the corresponding index by a
double index.

We shall use the perturbation theory [4]. We shall order the eigenvalues of problem (2.2) in the usual
way. The first eigenvalue of problem (2.2) splits, in general, into \( l_1 \) different eigenvalues (1.2) at \( k = 1 \):

\[
0 < \Lambda_{11} = \Lambda_{21} = \ldots = \Lambda_{i_1,1} < \Lambda_{i_1+1,1} \leq \ldots; \quad (B_{1i} \Phi_{1i}, \Phi_{1i}) = \eta_{i_1};
\] (2.4)

\[
(A_{10} \Phi_{1i}, \Phi_{1i}) = \Lambda_{1i} \eta_{i_1}; \quad \lambda_{1i} = \Lambda_{1i} + \delta_i \Lambda_{i1}^{(1)} + \delta_i^2 \Lambda_{i1}^{(2)} + \ldots \quad (i = 1, 2, \ldots, l_1).
\]

Here \( \Phi_{1i} \) are the orthonormalized eigenfunctions of problem (2.2). It is most interesting to find the value \( l_1 \),
and it is of secondary importance to construct the actual form of the operators \( A_{1i}, B_{1i} \) [see (2.3)]. The mul-
tiplicity of the first eigenvalue is sometimes very sensitive to perturbations (variations) of the parameter \( \delta_i \):

\[
l_1 = l_1(\epsilon, \delta_i) \text{ (sometimes } l_1 \to \infty \text{ as } \epsilon \to 0).\] (2.5)

The results of the calculations with regard to the determination of \( l_1 \) are given for specific problems in
Sec. 4.

The modified problems of type (2.2) are classified according to the multiplicity of the first eigenvalue in
the following way: 1) \( l_1 = 1 \), when the spectrum of problem (1.2) is widely spaced in the vicinity of \( \lambda_{1i} \); 2) \( l_1 \gg 1 \),
when the spectrum in the vicinity is dense; for example, the multiplicity of the first eigenvalue of problem (2.2)
reaches several tens of hundreds [2, 5] when the spectrum begins at the accumulation point.

The solution of problem (2.1) is sought in the form of an asymptotic series in the parameter \( \delta_i \) (the limit-
ning problem is found in the spectrum [6]):

\[
F(x, y, 0) = \delta_i^{-1} \sum_{i=1}^{l_1} C_{0i} \Phi_{0i} + \left( F_{0i} + \sum_{i=1}^{l_1} C_{0i} \Phi_{0i} \right) \]

\[
+ \delta_i \left( F_{1i} + \sum_{i=1}^{l_1} C_{1i} \Phi_{1i} \right) + \delta_i^2 \left( F_{2i} + \sum_{i=1}^{l_1} C_{2i} \Phi_{1i} \right) + \ldots, \quad \delta_i \ll 1.
\] (2.6)

Here \( \Phi_{0i}, \Phi_{1i}, \ldots, \Phi_{li} \) are the eigenfunctions of problem (2.2), while the number \( l_1 \) is first specially selected
[see (1.5) and (2.5)]. By substituting the representation (2.6) of the solution into the equation and boundary
conditions of problem (2.1), we obtain

\[
\{(A_{0i} + \delta_i A_{1i}) - [\Lambda_{1i} + \delta_i (\Lambda_{1i}^{(1)} - \lambda_{1i}^{(1)}) + \delta_i^2 \Lambda_{1i}^{(2)} + \ldots](B_{0i} + \delta_i B_{1i})\}
\]

\[
\times \left[ \delta_i^{-1} \sum_{i=1}^{l_1} C_{0i} \Phi_{0i} + \left( F_{0i} + \sum_{i=1}^{l_1} C_{0i} \Phi_{0i} \right) + \ldots \right] = Z;
\] (2.7)