ELASTIC CHARACTERISTICS OF POROUS MATERIALS

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Investigations of the characteristics of matter under extreme conditions of impact-wave loading have incited considerable interest in the analysis of the elastic characteristics of porous materials [1]. The importance of investigating the elastic wave velocity and the elastic moduli of porous media is also seen in designing new composite materials suitable for operation under conditions of high pressure and temperature and variable mechanical fields [2, 3]. The characteristics of porous solids are usually described within the framework of the mechanics of continuous media by devising geometric structure models [4]. We shall investigate here the elastic characteristics of porous materials by using the semi-empirical Morse equation for the energy of a strained solid.

Using the intrinsic energy as a function of the volume \( V \), assigned by the Morse equation [5], we write the adiabatic modulus of cubic compressibility \( B \) in the following form:

\[
B = \frac{B_0}{\alpha} x^{1/3} \left( 2 \left( x^{1/3} + \alpha \right) \exp 2\alpha \left( 1 - x^{1/3} \right) - \left( 2x^{1/3} + \alpha \right) \exp \alpha \left( 1 - x^{1/3} \right) \right),
\]

where \( x = V_0/V \); \( B_0 \) and \( V_0 \) are the modulus and the specific volume of the free material, respectively. The \( \alpha \) parameter, which characterizes the forces of interatomic repulsion and attraction, was determined in [5] with respect to the energy and the modulus \( B \) in the unstrained state. According to the proposed approach, the elastic characteristics of a porous solid were considered as the parameters of the effective medium obtained as a result of cubic expansion of a free, compact body by repulsive forces to a density corresponding to a porous material. Introducing the porosity defined by \( P = (V - V_0)/V \), we have the following expression for the elastic modulus:

\[
B(P) = \frac{B_0}{1 + \alpha} \left( y + \alpha \right) \exp 2\alpha \left( 1 - 1/y \right)
\]

(\( y = 1 - P \)). Equation (1) makes it possible to determine the expression for the volume wave velocity \( C_w \):

\[
C_w(P) = C_{w0} \left( \frac{y + \alpha}{(1 + \alpha)y} \right)^{1/2} \exp 2\alpha \left( 1 - 1/y \right)
\]

(\( C_{w0} \) is the value of \( C_w \) for \( P = 0 \)). It should be noted that \( C_w \) is of interest as the first term in the expression of the impact wave velocity with respect to the mass velocity in the material [6]. Equations (1) and (2) were used for calculating the velocity \( C_w \) and the modulus \( B \) for porous iron. The values of the parameter \( \alpha \), borrowed from [5], and of the modulus \( B_0 \), borrowed from [7], that were used in calculations are given in Table 1. The calculation results are represented by the curves in Fig. 1b and 2b.

In order to check the adequacy of this approach, we performed ultrasound measurements of the longitudinal \( C_L \) and the transverse \( C_T \) velocities of elastic waves. The test specimens were cylindrical and had a diameter of 10 mm and a length of 15 mm. They were made of PZhRV2 iron powder with a mean particle size of 81 \( \mu m \) by pressing and sintering in vacuum at 1450 K. The porosity of the specimens was specified in the range from 4 to 45%. The mean dimension \( a \) of the pores, which emerged as the basic scattering centers, was measured by means of a NEOPHOT-32 optical microscope and was monotonically increased from \( a = 7 \mu m \) for \( P = 5% \) to \( a = 16 \mu m \) for \( P = 40% \).
TABLE 1

<table>
<thead>
<tr>
<th>Material</th>
<th>c</th>
<th>$v_0$</th>
<th>$\varepsilon_0$</th>
<th>$p_0$</th>
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<tr>
<td>MgO</td>
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<td>159</td>
<td>307</td>
<td>130</td>
</tr>
</tbody>
</table>

The measurements were performed according to the phase interference method with separately matched piezoelectric transducers [8]. The ultrasound velocity was determined by automated analysis of the phase shift between the emitted and the received signals. The frequency of the ultrasound signal was equal to \( f = 2.5 \text{ MHz} \), which corresponded to the case \( ka = (2\pi f/C) a << 1 \). For specimens of compact Armco iron (\( P = 0 \)), the error in measuring \( C_{v0} \) and \( C_{l0} \) was less than 1%.

As the porosity increases, considerable dispersion of elastic waves is possible due to the increasing size and concentration of pores. The relative contribution of the dispersion to the ultrasound velocity was estimated to be \( C_{v0} P (ka)^2/C_{l0} \) in accordance with [9]. The experimental values of this contribution varied from 0.002% for \( P = 5\% \) to 0.35% for \( P = 40\% \).

The measured ultrasound wave velocities are given in Fig. 1a (points 1), which also shows the experimental data: 2 [10] and 3 [11]. The velocity of volume waves was determined by means of the equation

\[
C_{w} = \left(C_{l}^2 - \frac{4C_{l}^2}{3}\right)^{1/2}
\]

(Fig. 1b). A monotonic reduction in the velocity with an increase in porosity was observed.

The measured ultrasound velocity made it possible to determine the elastic characteristics of porous iron as functions of \( P \). The experimental values of the modulus of cubic compressibility \( B \), Young's modulus \( E \), and the shear modulus \( \mu \) were determined by means of the expressions

\[
B = \frac{(C_{l}^2 - 4C_{l}^2)/3}{V}, \quad E = \frac{(3C_{l}^2 - 4C_{l}^2)}{C_{l}^2/(C_{l}^2 - C_{l}^2)} V, \quad \mu = C_{l}^2/V.
\]

The measurement results are given in Fig. 2 (points 1), which also shows the experimental data: 2 [12]; 3 [13]; 4 [14]; 5 [15].