This paper is devoted to the numerical modeling of the characteristics and structure of a new (for supersonic jets) class of disturbances and some applications of the results to the interpretation of the experimental data presented in [1-3]. The object of this paper is to obtain basic data and to verify the model of the existence of the Taylor-Görtler instability in a free supersonic flow. The literature available to us does not contain any information about such investigations. This work is a detailed exposition of the results presented briefly in [3, 4].

We consider first the hypothesis that the flow contains stationary rotational disturbances of the type Taylor-Görtler waves (T-G), excited by additional centrifugal forces arising due to the curvature of the trajectories of the gas following the real cellular "barrel-shape" structure of a nonisobaric jet. This choice, from among existing alternative choices, one of which is described in [5], is dictated by the following circumstances. First, longitudinal bands, indicating the existence of azimuthal nonuniformities of the optical density, are recorded near the nozzle cutoff, where the trajectories of the gas are actually curvilinear. Second, under the experimental conditions (underexpanded jet with degree of underexpansion N ~ 5) a wide-band spectrum of noise is recorded; this precludes the appearance of strong nonlinear effects at such early stages, and the other nonlinearities will be second-order infinitesimals compared to the linear T-G waves. The weak effect of sharp gradients at the nozzle cutoff flow discontinuities is indicated by the fact that the intensity of the bands decreases with increasing surface smoothness under constant efflux conditions.

So, the hypothesis that stationary rotational disturbances exist in the initial section of the jet is most plausible. Within the framework of this hypothesis we performed numerical modeling of the characteristics of the waves, studied the dependences on the flow parameters, and analyzed the experimental data in order to retrieve the local values of the density and velocity of the flow.

1. Equations for the Disturbances. The flow scheme within the first cell ("barrel") of the jet is displayed in Fig. 1. A system of linearized equations for T-G-wave-type disturbances, which includes a number of assumptions to be discussed below, was constructed in [5]:

\[\begin{align*}
U\omega' - 2Uu'/R_o + p'/\rho_0 &= 0, \\
U\omega' + p'/\rho_0 &= 0, \\
Uu' + U\omega' + [Uu'/R_o] + p'/\rho_0 &= 0, \\
U(p'/a^2 - \rho') - \rho p\omega' &= 0, \\
U\rho'_0 + p_0\omega' + \rho_0 u'_0 + \omega'_0 + u'/r + \omega'/r + [u'/R_o] &= 0.
\end{align*}\]  

This system was constructed for a one-dimensional flow of a compressible, nonviscous, heat-conducting gas with the velocity field \(u = |v', \omega', U + u'|\), where \(v', \omega', u'\) are, respectively, the transverse, azimuthal, and longitudinal, components of the disturbances in the coordinates \(r, \varphi, x\); \(\rho'\) and \(p'\) are the disturbances of the density and pressure, respectively; \(U = U(r)\) is the longitudinal component of the average velocity; \(\rho_0\) is the...
average flow density; and, $a$ is the local sound speed. The equations were represented in dimensionless form by scaling the variables to the following characteristic quantities: $r_0$ is the radial coordinate where the velocity is half its maximum value ($U = 0.5$); $U_0$, $\rho_0$ are, respectively, the maximum velocity and the density in the initial mixing layer.

The additional forces, which are proportional to $1/R_0$ ($R_0$ is the radius of curvature of the trajectories of the gas), are enclosed in brackets. The value of $R_0$ determines the centrifugal effects and the associated T-G waves. The system (1.1) is valid as a local approximation under the condition $R_0 = \text{const}$. As one can see from Fig. 1, $R_0$ changes in magnitude due to the growth of the boundary layer and the corresponding arrangement of the streamlines within the first "barrel." Determining the value of $R_0$ is itself a problem, which did not arise for flows near walls. The first and natural variant is to determine $R_0$ from sharp visible or measured reference points, such as the position of the suspended shock (SS) or the line of maximum total pressure $p_0$ in the compressed layer. This is apparently not entirely correct. Gas from the surrounding flooded space plays in the jet the role of a solid wall, on which all disturbances stop, and although the oscillations are mainly concentrated in the mixing layer, they can penetrate quite far into this space, thereby perturbing it. The values of $R_0$ calculated along the lines of asymptotic decay of disturbances differ significantly from the values of $R_0$ obtained from the exact reference points, and since the concept of radius of curvature is basic for analytical descriptions of T-G waves, an attempt must be made to find reasonable agreement between these values.

2. Average Flow. We study the problem in the plane-parallel approximation. It is acknowledged that in order to describe T-G waves in subsonic flows near walls the transverse component of the average velocity $V$ must be included in the calculations [6]; doing so ensures that diffusion and viscous effects are taken into account correctly. For a nonisobaric supersonic jet this component must be neglected at this stage due to the fact that there are no reliable data on the form and magnitude of $V$ on the initial section of the jet [7]. The profile of the longitudinal component $U$ is taken, just as in [5], from experimental approximations [8]. Doing so, of course, introduces into the results errors that cannot yet be estimated.

The compressed layer itself ($r_1 - r_3$ in Fig. 1) consists of two subregions [9]. In the first subregion, from the suspended shock $r_1$, the total pressure is restored up to its maximum value on the line $r_2$. This value is used as the start of the mixing layer (its inner boundary). Next, $\rho_0$ and therefore the average velocity $U$ decrease to their values in the flooded space ($\rho_0 \sim \rho_{out}$, $U \approx 0$). The coordinate $r_3$ is the conventional exterior boundary of the mixing layer, whose thickness $\delta = r_3 - r_2$. The half-velocity point virtually coincides with the half-width $\delta$ of the layer, so that $r_2 = 1 - \delta/2$ and $r_3 \geq 1 + \delta/2$, and $U(r_3) \approx 0.06$. Thus

$$U(r) = \begin{cases} 1 & r < r_2, \\ \exp(-0.693\eta^2) & r \geq r_2, \end{cases}$$

where the self-similar coordinate $\eta = 2(r - r_2)/\delta$. The relation between the average density $\rho_0$ and $U$ was determined from the gas-dynamic relation $\rho_0 = (1 + M_0^2(1 - U^2)(\kappa - 1)/2)^{-1}$, where $\kappa = c_p/c_v$ ($M_0$ is the Mach number with $U = U_0$). A modification of Sutherland's formula [8] can also be employed; this is virtually equivalent. The sound speed $a = (\rho_0M_0^2)^{-1/2}$.

In this work the structure of the acceleration section $r_1 < r < r_2$ was neglected, since it was assumed that in regions with positive velocities gradients $dU^2/dr > 0$, in accordance with the results of [6], the flow is more stable against T-G perturbations than in regions with $dU^2/dr = 0$.

3. Form and Eigenvalues of T-G Waves. The solutions of the system (1.1) can be obtained by two methods. In the first method, the linearized equations are directly integrated