EFFECT OF RHEOLOGICAL FACTORS ON THE LAWS OF MOTION AND HEAT TRANSFER OF VISCOELASTIC FLOWS AT LOW DEBORAH NUMBERS

A. N. Kekalov and V. I. Popov

The effect of structurally viscous and viscoelastic factors on the laws of motion and heat transfer in the entrance region of a channel are analyzed for the Deborah numbers.

The laws of motion and heat transfer of Newtonian fluids in the entrance region of channels have been thoroughly studied. Results in good agreement with experiment have been obtained by approximate and exact methods of solution [1].

The motion of viscoelastic fluids in the entrance region of channels has been treated in a number of papers. The main problem is the explanation of the effect of the rheological factors of fluids (the nonlinearity of the flow curve and the value of the reversible elastic deformation) on the pressure losses, the velocity distribution, the entrance lengths, etc.

Experiments with viscoelastic solutions of various concentrations for Reynolds numbers from 6 to 2000 led Sylvester and Rosen [2] to the conclusion that the nonlinearity of the flow curve (the exponent n in a power law) and the value of the reversible elastic deformation γe have opposite effects on the pressure losses in the entrance region of a pipe.

Broclebank and Smith [3] used flow visualization to measure the velocity field in various cross sections of the entrance region and determined entrance lengths. They showed that entrance lengths for the flow of viscoelastic fluids are appreciably greater than for a Newtonian fluid. The entrance lengths increased with increasing elasticity of the solutions. Similar results follow from theoretical studies [4, 5].

Unfortunately, unanimity in these questions has not yet been achieved. Experiments [6] with viscoelastic solutions of various concentrations over a range of Reynolds numbers from 1 to 270 showed that entrance lengths were 10-100% shorter than those obtained with inelastic fluids for the same power-law parameters.

Tandon [7] for flat channels and Bilgen [8] for circular pipes, using approximate methods of boundary-layer theory, also concluded that the entrance length decreases with increasing elasticity.

In the present paper we use boundary-layer theory methods to investigate the effect of rheological factors (structurally viscous β0 and viscoelastic γe) on the laws of motion and heat transfer of a fluid in the entrance region of a channel for the following velocity field.

\[ u = u(x, y), \quad v = v(x, y), \quad w = 0. \] (1)

In developing flows of viscoelastic fields, including those in the entrance region of channels, the first difference of the normal stresses \( b_{xx} - b_{yy} \) is different from zero. Therefore, with the usual approximations of boundary-layer theory [1], the equations of motion of a viscoelastic fluid can be written in the form

\[ \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial t_{xx}}{\partial x} + \frac{\partial t_{xy}}{\partial y}, \]

\[ 0 = -\frac{\partial p}{\partial y} + \frac{\partial t_{yy}}{\partial y}. \] (2)

Since \( t_{xx} - t_{yy} \) are zero in the flow core, Eqs. (1) reduce to the form

\[ \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - U \frac{\partial U}{\partial x} \right) = \frac{\partial}{\partial x} (t_{xx} - t_{yy}) + \frac{\partial \tau}{\partial y}. \] (3)

Here \( \rho U (\partial U / \partial x) = (\partial t_{yy} / \partial x) \). Equation (3) is the starting point for the derivation of the integral momentum equation. Omitting terms known for ordinary Newtonian fluids, we obtain finally

\[ \rho \left[ U \frac{\partial U}{\partial x} \left( 2\delta* + \delta^{\ast} \right) + U^2 \frac{\partial \delta**}{\partial x} \right] = \tau_w (x) - \int_0^\delta \frac{\partial}{\partial x} (t_{xx} - t_{yy}) dy. \] (4)

Here

\[ \delta* = \frac{1}{U} \int_0^\delta (U - u) dy, \quad \delta^{\ast} = \frac{1}{U^2} \int_0^\delta u (U - u) dy. \]

In the stabilized flow region the velocity profile of a viscoelastic fluid obeying a linear fluidity law has the form

\[ \omega \equiv \frac{u}{U_{\text{max}}} = A \left[ 2\xi - \xi^2 + B (3\xi - \xi^2 + \xi^3) \right], \quad A = (1 + B)^{-1}, \]

\[ B = \frac{4}{9} \left[ \left( 1 + 18 \frac{\beta_0}{Re_0} \right)^{0.5} - 1 \right], \quad \xi = \frac{y}{b}, \quad \beta_0 = \frac{\theta}{v_b} \rho V^2. \]

Therefore, we approximate the velocity profile in the entrance region by a cubic polynomial

\[ \omega \equiv u/U (x) = a_0 + a_1 k + a_2 k^2 + a_3 k^3, \quad k = y/b. \]

We find the coefficients from the boundary conditions \( \omega = 0 \) for \( k = 0 \); \( \omega = 1 \), \( d\omega / dk = 0 \) for \( k = 1 \):

\[ \int_0^1 \omega dk \bigg|_{k=x}^{k=b} = \int_0^1 \omega dk, \]

Starting from these conditions, it can be shown that \( a_0 = 0 \), \( a_1 = -6 + 8A + 9AB \), \( a_2 = 15 - 16A - 18AB \), and \( a_3 = -8 + 8A + 9AB \). Therefore,

\[ \delta* = \delta \int_0^1 \left( 1 - \frac{u}{U (x)} \right) dk = A_0 \delta, \] (7)

\[ \delta^{\ast} = \delta \int_0^1 \frac{u}{U (x)} \left( 1 - \frac{u}{U (x)} \right) dk = B_0 \delta, \] (8)

\[ A_0 = 1 - \frac{a_1}{2} - \frac{a_2}{3} - \frac{a_3}{4}, \]

\[ B_0 = \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} - \frac{a_2^2}{3} - \frac{a_3^2}{5} - \frac{a_2 a_3}{2} - \frac{a_3 a_2}{3} - \frac{2a_2 a_3}{5}. \]

From the condition for a constant flow rate in a flat channel, assuming that the velocity profile is uniform at entry,

\[ \delta \int_0^\delta u dy + U (x) (b - \delta) = Vb \]

we have

\[ U (x)/V = (1 - A_0 \Delta)^{-1}, \quad \Delta = \delta (x)/b. \] (9)

Using the fundamental rule for differentiating under the integral sign, and taking account of the fact that in the region where the boundary layer and the flow core join \( t_{xx} - t_{yy} \approx 0 \), Eq. (4) can be written in the form

\[ \rho \left[ U \frac{\partial U}{\partial x} \left( 2\delta* + \delta^{\ast} \right) + U^2 \frac{\partial \delta**}{\partial x} \right] = \tau_w (x) - \frac{\partial}{\partial x} \int_0^\delta (t_{xx} - t_{yy}) dy. \] (10)