ADVANTAGES OF THE DILATOMETER INVESTIGATION
OF THE HEAT-TRANSFER COEFFICIENT OF A
DIELECTRIC LAYER

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A dilatometer method for the comparative determination of the heat-transfer coefficient
of dielectrics which is fast, accurate, and simple is described.

When using rapid dilatometry to determine such important thermal characteristics as the heat-transfer
coefficient $k$ of a dielectric layer, the method not only possesses high accuracy, simplicity of the apparatus
design, and the possibility of recording the kinetic characteristics of the thermal expansion of the objects being
investigated in arbitrary scale units [1, 3], but also, which is no less important, the complete absence of the
need to measure the temperature or the thermal fluxes in these specimens [4] and, to an equal extent, of the
layer thickness $h$ (on which, like the thermal conductivity of the material of the layer $\lambda$, depends the quantity
$k = \lambda / h$ [5]). This fact is particularly convenient when studying free-flowing and paste-like materials.

One of the fundamental theoretical assumptions which makes dilatometry attractive for the "nontemper-
ature" determination of the coefficient $k$ of a dielectric layer is the behavior of the comparative thermal ex-
pansion of two semiinfinite prismatic bodies with a common initial constant temperature $T_1$. Heating (or cool-
ing) of the specimens occurs due to sudden simultaneous contact of their fixed bases with a heat carrier (a
coolant) at a fixed temperature $T_2$ for a heat-exchange factor which is the same for both ends, ensuring uni-
form heat transfer in both specimens, the side surface of which is heat insulated, and one is justified using
the laws of heat transfer in a uniform infinite half-space.

It is easy to see that the same behavior also holds for bodies in the initial stage of the propagation of a
thermal disturbance in objects of finite size when the opposite heated end has an initial temperature $T_1$ [6].
The velocity of motion of this end $W$ is proportional to the thermal expansion coefficient $\beta$, its thermal dif-
fusivity $a$, and the gradient $(\partial T / \partial x)(0, t)$ at the heated end [6];

$$W = \beta \frac{\partial T}{\partial x} (0, t).$$

(1)

The temperature gradient at the fixed end $(\partial T / \partial x) (0, t)$ immediately after contact with the thermal
carrier flow depends on the parameters $a$, $\lambda$, and $a$ [7];

$$\frac{\partial T}{\partial x} (0, t) = \frac{\alpha}{\lambda} (T_2 - T_1) \exp \left( \frac{\alpha^2}{\lambda^2} a t \right) \left[ 1 - \varphi \left( \frac{\alpha}{\lambda} \sqrt{at} \right) \right],$$

(2)

where $\varphi(z)$ immediately after contact with the thermal carrier flow depends on the parameters $z = (\alpha / \lambda) \sqrt{at}$.

It is easy to show that the rate of expansion $W_0$ when $t = 0$ (at the initial instants of the process) is inde-
dependent of the time $t$:

$$W_0 = \beta \frac{\partial T}{\partial x} \frac{a}{\lambda}.$$

(3)

If one of the specimens (the characteristics of which are exactly the same as the characteristics of a
standard cylinder) is placed in the same part of the channel for the flow of heat carrier, but the heated end of
which is separated from the heat carrier by the plane-parallel layer being investigated, the rate of expansion

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Fig. 1. Schematic graphic calculation of the initial velocities of thermal expansion of the standard specimen and the specimen being investigated; \( \Delta l_2(t) \) and \( \Delta l_1(t) \) are the kinetic thermal deformation curves of the standard specimen in the presence of the dielectric layer being investigated and when there is no dielectric layer; \( t_1 \) is the instant when the standard specimen reaches an expansion \( \Delta l_1 \); \( W_0 = \tan \angle AOC \); \( W_1 = \tan \angle FOC \); \( \Delta l_1 = DE \); \( (BC/AC) = 0.57 \).

of this cylinder at the initial instants of the process \( W_1 \) will differ analytically from Eq. (3) by a factor which contains, instead of the coefficient \( \alpha \), a certain effective parameter \( \alpha_1 \):

\[
W_1 = \alpha \beta (T_2 - T_1) \frac{\alpha}{\lambda}.
\]  

(4)

The value of the effective heat-transfer coefficient \( \alpha_1 \) is determined, following the well-known rule for the addition of thermal resistances \([5]\), in terms of the required characteristic of the layer \( k \) and the heat-transfer coefficient \( \alpha \):

\[
\frac{1}{\alpha_1} = \frac{1}{k} + \frac{1}{\alpha}.
\]  

(5)

Substituting the value of \( \alpha_1 \) from Eq. (5) into Eq. (4) and dividing the equation obtained by Eq. (3), we have

\[
\frac{W_1}{W_0} = \frac{k}{\alpha + k}.
\]  

(6)

It will be convenient henceforth to eliminate the heat-transfer coefficient \( \alpha \). To do this we express the velocity (1) in terms of the gradient (2) and integrate it with respect to time over the expansion \( \Delta l(t) \):

\[
\Delta l(t) = \beta (T_2 - T_1) \frac{\lambda}{\alpha} \left[ \exp \left( \frac{\alpha^2}{\lambda^2} at \right) \left( 1 - \varphi \left( \frac{\alpha}{\lambda} \sqrt{\frac{a}{t}} \right) \right) - 1 \right] + \frac{2}{\sqrt{\pi}} \frac{\alpha}{\lambda} \sqrt{at}.
\]  

(7)

It is easy to show by direct substitution that the expression in square brackets of Eq. (7) for a value of the argument \( z_1 \) given by

\[
z_1 = \frac{\alpha}{\lambda} \sqrt{at_1} = 1
\]  

(8)

has a value of 0.57, and then, taking into account property (3), the expansion of the standard specimen \( \Delta l_1 \) from the time when the process starts up to the instant \( t_1 \), calculated from Eq. (8), reaches the values

\[
\Delta l_1 = \Delta l(t_1) = 0.57 \frac{\lambda^2}{\alpha a^2} W_0.
\]  

(9)

Multiplying the numerator and denominator of the fraction in Eq. (9) by the time factor \( t_1 \) and bearing in mind Eq. (8), we obtain

\[
\Delta l_1 = 0.57 W_0 t_1.
\]  

(10)

By solving Eq. (10) graphically it is easy to find the point \( t_1 \) at which the expansion \( \Delta l_1 \) satisfies condition (10). However, in practice, finding the instant \( t_1 \) is considerably simplified by the obvious fact that the ratio \( \Delta l_1/t_1 \) is nothing more than the slope of the section of line which connects the origin of coordinates and the experimental curve of the expansion \( \Delta l(t) \) of the standard cylinder without the layer being investigated (see Fig. 1). Consequently, if by definition the velocity \( W_0 \) is the slope of the curve \( \Delta l(t) \) at the origin of coordinates (see the figure), then, by choosing, using condition (10), the point B on the section AC such that the length