INVESTIGATION OF THE HEAT-TRANSFER CHARACTERISTICS IN DECOMPOSING HEAT-SHIELD MATERIALS

K. G. Omel'chenko, M. V. Savelov, and V. P. Timoshenko

A derivation is given for the general equations of heat and mass transfer in decomposing heat-shield material on the basis of using the Umov transfer equation. A simplified system of equations is presented which contains the functional thermophysical characteristics. The functional thermophysical characteristics are determined in a numerical example for the case when the functionality is due to thermal decomposition of the material according to the Arrhenius law.

At present, principally decomposing material of the bonded plastics type are used in heat shields operating under intensive thermal load conditions. Complex and multistage physicochemical transformations occur in these materials under high-temperature heating, which alter the structure, chemical composition, and physical properties of the material substantially, and exert significant influence on the heat transfer within the coating. Models based on using the mass-, energy-, and momentum-conservation laws with different simplifying assumptions [1-4] are ordinarily used to describe mathematically the heat- and mass-transfer processes in decomposing materials, and, as a rule, the questions of the practical determination of the physical characteristics used in these equations hence remain open.

On the basis of using the Umov equation, the derivation of the general heat- and mass-transfer equations in a decomposing heat-shield material is given in this paper. Simplified equations are presented which have been obtained by introducing the functional thermophysical characteristics. The method of determining the functional thermophysical characteristics, which is based on solving the inverse problem, is examined in the example of a numerical experiment.

Let us assume that the decomposing heat-shield material can be considered as a homogeneous porous medium consisting of a gaseous phase and a solid phase, where each phase consists of a definite number of mutually reacting components. To obtain the fundamental conservation equations, let us, analogously to [5], use the Umov equation of substance transfer,

\[
\frac{\partial C}{\partial t} + \text{div}(CV_C) = -\text{div}(JC) + I_C, \tag{1}
\]

where \( C, V_C, J_C, I_C \) are, respectively, the concentration, velocity of convective transfer, diffusion flux, and intensity of the volume sources of the substance.

The mass-conservation equation of the \( i \)-th gaseous component is obtained from (1) by using the following relationships:

\[
C = mp_{1i}, \quad V_C = V_1 \frac{s}{m}, \quad J_C = s_{1i}, \quad I_C = \omega_{1i}, \quad \frac{\partial}{\partial t} (mp_{1i}) + \text{div}(m_{1i}V_1) = \text{div}(s_{1i}) + \omega_{1i}. \tag{2}
\]
Summing (2) over all $i$-components and taking account of the relationships

$$\rho_i = \sum_{i=1}^{N} \rho_{ni}, \quad \sum_{i=1}^{N} j_{ni} = 0, \quad \omega_i = \sum_{i=1}^{N} \omega_{ni},$$

we obtain the mass-conservation equation for the gaseous phase:

$$\frac{\partial}{\partial t} (\rho_1 V_1) + \text{div} (\rho_1 V_1 V_1) = \omega_1. \quad (3)$$

The mass-conservation equation for the solid phase can be obtained analogously:

$$\frac{\partial}{\partial t} \left[ (1 - m) \rho_2 \right] + \text{div} \left[ (1 - s) \rho_2 V_1 \right] = - \text{div} \left[ (1 - s) j_{ni} \right] + \omega_{2i}. \quad (4)$$

$$\frac{\partial}{\partial t} \left[ (1 - m) \rho_2 \right] + \text{div} \left[ (1 - s) \rho_2 V_1 \right] = \omega_2. \quad (5)$$

$$\rho_2 = \sum_{i=N+1}^{N+M} \rho_{2i}, \quad \omega_2 = \sum_{i=N+1}^{N+M} \omega_{2i},$$

Here $V_2$ is the mean mass flow rate of the solid phase, the velocity of material shrinkage, or deformation because of the thermal stresses, pressure, external forces, etc.

To derive the momentum-conservation equation of the gas phase, let us use the relationships

$$C = m \rho_1 V_1, \quad V_C = \frac{s_1}{m} V_1, \quad j_c = s_1 p, \quad I_c = m \rho_1 F_1, \quad (6)$$

where $p$ is the pressure, $s_1$ is a coefficient [6] reflecting the magnitude of the surface through which the pressure pulse is transmitted (for the case when the porous material is a system of linear capillaries $s_1 = s$), and $F_1$ is the fictitious mass resistance force during gas filtration in the porous medium.

The equation of solid-phase motion cannot be written down, since the processes associated with displacement of the solid carcass, including the shrinkage, have still been investigated only slightly. Hence, we shall consider the velocity $V_2$ a given quantity.

Let us use the equation of state in the form

$$p = \rho_1 RT_1 \quad (7)$$

to obtain the connection between the pressure and density of the gas phase. The energy-conservation equation for the gas phase is obtained from (1) with the following relationships taken into account:

$$C = m \rho_1 \left( U + \frac{1}{2} V_1 V_1 \right), \quad V_C = \frac{s_1}{m} V_1, \quad h = U + \frac{p}{\rho} = \int_{0}^{T} c_p dT, \quad (8)$$

$$I_c = - s_1 \frac{\partial}{\partial t} T_1 + s \sum_{i=1}^{N} h_{1i} T_{1i} + H_{R,1} + s_1 p V_1,$$

$$I_c = \alpha (T_s - T_1) + m \rho_1 F_1 V_1 + \sum_{i=1}^{N} h_{1i}^\ast \omega_{1i},$$

$$\frac{\partial}{\partial t} \left[ m \rho_1 \left( h_i + \frac{1}{2} V_i V_i \right) \right] + \text{div} \left[ \rho_1 \left( h_i + \frac{1}{2} V_i V_i \right) V_i \right] =$$

$$= \frac{\partial}{\partial t} (mp) + \text{div} (s_1 \frac{\partial}{\partial t} T_1) - \text{div} \left( s \sum_{i=1}^{N} h_{1i} T_{1i} \right) -$$
$$- \text{div} (H_{R,1}) + \alpha (T_s - T_1) + m \rho_1 F_1 V_1 + \sum_{i=1}^{N} h_{1i}^\ast \omega_{1i},$$

where $H_R$ is the integrated radiant energy flux (relative to the wavelengths), $\alpha$ is the volume coefficient of heat exchange between the solid and gas phases, and $h_{1i}^\ast$ is the enthalpy of the $i$-th gaseous component at the temperature $T_2$. 889