The diameter of the bottoms at the equator was 400 mm. The shells lost rigidity along a ring 30-60 mm wide in the region with \( R_1 = 112 \text{ mm} \) and \( R_2 = 224 \text{ mm} \). As is seen from Table 1, the values estimated from the equation are 32-39\% higher than the critical pressures obtained experimentally; this shows satisfactory agreement.

LITERATURE CITED

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OPTIMIZATION OF THREE-LAYERED PLATES OF HYBRID COMPOSITE MATERIALS

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With the objective of improving the properties of elements of constructions, in recent times investigators have turned to multilayered materials for the construction of which combinations of fibers with various physicomechanical properties are used, or to the so-called hybrid composite materials. In the given paper we consider optimization of three-layered plates formed of composites with various properties of the reinforcement and with different structure, and also in combination with layers of light alloys.

We shall consider a thin three-layered plate elongated in the direction of action of a compressive force, of width \( B \), length \( L \) (\( L \gg B \)), of overall thickness \( h \); the thickness of its middle layer is \( h_1 \), the thickness of the outer layers being \( h_2/2 \) (Fig. 1). The critical force of such a plate, formed of stiff orthotropic layers, according to the hypothesis of nondeformable normals is given by the relation [1, 2]

\[
P_{cr} = \frac{\pi h}{6B} P,
\]

where \( D = \sqrt{D_{11}D_{22}} + D_{12} + 2D_{66}; D_{11} = A_{1111}H + A_{1122}(1 - \nu)^3; D_{22} = A_{2222}H + A_{2222}(1 - \nu)^3; D_{12} = A_{1212}H + A_{1212}(1 - \nu)^3; D_{66} = A_{1212}H + A_{1212}(1 - \nu)^3; H = \mu^3 + 3\nu^2(1 - \nu) + 3(1 - 2\nu)^2; \nu = h_2/h.\)

The quantities \( A_{ijkl} \) and \( A^*_{ijkl} \) are determined according to [1] from the components of the flexibility tensor \( a_{ijkl} \) of the composite according to the relations \( A_{1111} = a_{1111}/\Delta; A_{1122} = -a_{1122}/\Delta; A_{2222} = a_{1111}/\Delta; A_{1212} = (1/\Delta)a_{1212}; \Delta = a_{1111}a_{2222} - a_{1212}^2. \)

In the optimization the following composites were considered: boron-reinforced plastic, glass-reinforced plastic and organic-fiber plastic. The physicomechanical properties of the materials have been taken from the literature [4-6] and presented in Tables 1 and 2. The properties of the alloy D16-T are taken according to [7, 8], with the components of the flexibility tensor at the instant of buckling taken in accordance with the theory of local strains:

\[
a_{1111} = 1.39 \cdot 10^{-6} + 2.45 \cdot 10^{-6} a_{1111}^{all}; \quad a_{2222} = 1.39 \cdot 10^{-6} + 0.6225 \cdot 10^{-6} a_{1111}^{all};
\]

\[
a_{1212} = 2.085 \cdot 10^{-6} + 0.0225 \cdot 10^{-6} a_{1111}^{all}; \quad a_{1122} = -0.5 a_{1111}^{all}.\]
We consider first the dependence of the critical stress on the structure of reinforcement of a plate, homogeneous with respect to height, in an elastic formulation. The width of the plate, B, is taken equal to 1 m, while the thickness h is taken as 1 cm. Three composites with up to five directions of reinforcement were investigated; at the same time it was established that for all the materials considered the structure of reinforcement according to the scheme $\pm 45^\circ$ corresponds to the maximum critical stress. It is interesting to note that, according to the data of Table 3, chaotically reinforced material in a plane and even in space gives larger values of $\sigma_{11cr}$ than the material unidirectionally reinforced in the direction of the acting force.

Next we consider the problem of optimization of a plate. As the optimality criterion we take the minimum of the mass:

$$G = \left( \frac{h_1}{\gamma_1} + \frac{h_2}{\gamma_2} \right) B L_0, \quad L_0 = 1 \text{ m},$$

where $\gamma_1$, $\gamma_2$ are the specific masses of the plate layers. As the constraints we take a given value of the critical load and the known limits of strength of the layer. The parameters to be optimized are the thickness of each layer and the angles of reinforcement $\theta_i$. The problem is solved with the use of the algorithm [9].

The three-layered plate (see Fig. 1) was composed of four materials — glass-reinforced plastic, boron-reinforced plastic, organic-fiber plastic, and alloy D16-T. Theoretically 12 arrangements of materials in a symmetrical three-layered plate are possible. However, in the process of optimization in an elastic formulation it appeared that only three of them yield a gain with respect to the mass incomparison with a single-layered plate made of a single material of the pair of materials being considered. As seen from the data of Table 3, the scheme of reinforcement $\pm 45^\circ$ is optimal both for the outer and the inner layer. The data in Table 4 show that the most advantageous pair of materials is organic-fiber plastic (inner layer) and boron-reinforced plastic. For comparison purposes we have presented in Table 4 data on optimization of a plate in the case of a given structure of reinforcement of the layer (isotropic in the plane of reinforcement).

The results of the calculation of plates, homogeneous over the height, with the same relative reinforcement content (according to the data of Table 4) for three structures of reinforcement are presented in Table 5. As was to be expected, plates that are homogeneous over the height are less advantageous. It is interesting to note that for plates reinforced by two forms of reinforcement, even in the case of a structure that is homogeneous over the height, an optimal relation of the amount of these fibers, resulting in a minimum mass, is possible. Qualitatively this phenomenon is seen from Fig. 2, where we have presented the curve of the plate mass with different relations of organic boron fibers. We see that the minimum of the mass in the given case corresponds to 80% organic and 20% boron fibers, for the overall reinforcement coefficient $\mu = 0.6$. In the given case a small addition of boron fibers more substantially varies the stiffness properties of the material than its mass.