HYDRODYNAMICS AND HEAT TRANSFER IN A LAYER
OF LIQUID ON A ROTATING SURFACE, ALLOWING
FOR INTERACTION WITH A GAS FLOW

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The hydrodynamics and heat transfer of a layer of liquid on a rotating surface are analyzed theoretically on the boundary-layer approximation under conditions in which a gas flow interacts with the liquid film.

The hydrodynamics and mass transfer of a layer of liquid on a rotating surface were studied in the absence of wave formation and frictional forces at the interface in our earlier paper [1]. In this paper the same problem will be solved with due allowance for frictional forces at the interface, and the heat transfer from the liquid film to the rotating surface will be calculated under these conditions.

1. Let the x axis signify the arc length along the flooded wall of a spiral channel and y, the distance from the wall along the normal. We assume that the liquid is incompressible, the motion steady, and the flows isothermal. The thin layer of liquid moves without wave formation along an Archimedes spiral, which in polar coordinates r, $\theta$ obeys the equation $r = \Lambda \theta$, $\Lambda > 0$. We also assume that the pressure gradient in the layer arises solely from the rotation. Under these assumptions the motion of the thin layer of liquid may be described by the same Prandtl equations as in [1]:

$$\begin{align*}
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= X - \frac{1}{\rho} \frac{\partial p}{\partial x} + y \frac{\partial u}{\partial y}, \\
\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= Y - \frac{1}{\rho} \frac{\partial p}{\partial y}, \\
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} &= 0;
\end{align*}$$

(1)

$R(x)$ is the radius of curvature in polar coordinates:

$$R(x) = \frac{(r^2 - r_0^2)^{3/2}}{r^2 + 2r_0^2 - rr_0^2} = \frac{\Lambda (\theta^2 + 1)^{3/2}}{\theta^2 + 2},$$

where

$$r_0 = \frac{dr}{d\theta}, \quad r_{\theta \theta} = \frac{d^2r}{d\theta^2};$$

and $X$, $Y$ are the projections of the mass forces on the $x$ and $y$ axes respectively. The mass forces acting on the particles of liquid include the centrifugal force $F_c = \omega^2 R(x)$ and the Coriolis force of inertia $F_{\text{cor}} = 2\omega \times \bar{V}$. A change in the direction of rotation of the spiral is only reflected in the second of these. The projections of the mass forces on the $x$ and $y$ axes take the form

$$\begin{align*}
x &= \omega^2 R(x) \cos \alpha \pm 2\omega v, \\
y &= \omega^2 R(x) \sin \alpha \pm 2au,
\end{align*}$$

(2)

where the upper sign corresponds to the anticlockwise, and the lower sign to the clockwise, rotation of the spiral; $\alpha$ is the angle made by the centrifugal force vector with the positive direction of the tangent, since
In the variables \( \theta, y \) the system of Eqs. (1) may be expressed as follows:

\[
\frac{u}{A \sqrt{\theta^2 + 3}} \cdot \frac{\partial u}{\partial \theta} + v \frac{\partial u}{\partial y} = X - \frac{1}{\rho} \cdot \frac{\partial p}{\partial \theta} + \gamma \frac{\partial^2 u}{\partial y^2},
\]

\[
- \frac{\nu^2}{R(\theta)} - \gamma \frac{1}{\rho} \frac{\partial p}{\partial y},
\]

\[
\frac{1}{A \sqrt{\theta^2 + 1}} \cdot \frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial y} = 0.
\]

It follows from the condition of attachment to the wall that

\[
\text{for } y = 0, \quad u = v = 0.
\]

The effect of the gas flow on the flow of the thin liquid layer is taken into account by way of the tangential forces on the interface, i.e.,

\[
\text{for } y = \delta, \quad \frac{du}{dy} = \frac{\tau \rho_0}{\mu} = B, \quad \rho = \rho_0 = \text{const}, \quad u = -U.
\]

We solve system (3) by the method of integral relationships. The polynomial of the second degree which satisfies boundary conditions (4) and (5) takes the form

\[
\frac{u}{U} = \left(2 - \frac{B \delta}{U} \right) \frac{y}{\delta} - \left(1 - \frac{B \delta}{U} \right) \left(\frac{y}{\delta}\right)^2,
\]

for \( \delta = 0.5 \), Re = 300; \( \Gamma_1 = 1; \ E_1 = 0.4 \); 1) 1.6; 2) 2.5; b) for \( E_1 = 1; \ E_5 = 0.5 \); 6) 1; 7) 1.5.

Fig. 2. Dimensionless thickness of the liquid film as a function of the length of the spiral for \( E_1 = 1; \ E_5 = 1 \): a) for \( \Gamma_1 = 1; \) Re = 100; 2) 300; 3) 500; 4) 1000; b) for \( \text{Re} = 300, \) \( \Gamma_1 = 2; \) 6) 1; 7) 0.

Fig. 3. Dimensionless thickness of the thermal boundary layer as a function of the length of the spiral for \( E_1 = 1; \ E_5 = 1; \) Pr = 10: a) for \( \Gamma_1 = 0; \) Re = 1000; 2) 500; 3) 300; 4) 100; b) for \( \text{Re} = 300, \) \( \Gamma_1 = 0; \) 6) 1; 7) 2.

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