Thermal dissociation of a polydisperse lump material

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The problem of the dissociation of a polydisperse lump material is examined using the statistical approach and making due allowance for temperature-dependence of the coefficient of thermal conductivity.

A number of papers [1-5] have been written on the subject of the process of thermal dissociation of materials in lump form. It has been shown experimentally [2] that there is a fairly clear interface between the dissociated and undissociated substance which runs deep into the lump. This interface is the surface at which the heat passing through the shell of reacted substance is consumed. The model of a heat exchanger with a variable heat-exchange surface [5] is, therefore, suitable for use as a physical model of the process which is compatible with the experiment.

A formula for the time required for the dissociation of a single lump under the conditions of constant thermophysical process characteristics is devised in [5] on the basis of several physically sound hypotheses which simplify the investigation. It is, however, a well-known fact [6-8] that the thermophysical characteristics of substances being heated are not constant, in particular, the coefficient of thermal conductivity can be described as a linear function of temperature:

\[ \lambda = \lambda_0 (1 + \omega t). \]  

(1)

It should be noted that if this relationship is disregarded for industrial furnace operating conditions, there will be significant errors in the calculation of the material dissociation time. In addition, when samples are heated, their porosity \( p \) is changed according to the relation [9, 10]

\[ p = 115.2 - 0.078 t. \]  

(2)

By definition

\[ p = \frac{\rho_1 - \rho_a}{\rho_1} \times 100. \]  

(3)

The dependence of the coefficient of thermal conductivity on the temperature and apparent volumetric mass of the material has been found in [7] in the form

\[ \lambda = 1.163 (1 - 1.011 \times 10^{-2} t + 1.513 \times 10^{-4} \rho_a). \]  

(4)
By combining (2)-(4) and taking into account the fact that the true volumetric mass of lime from limestone is \( p_L = 3360 \text{ kg/m}^3 \), it is possible to find values for the coefficients involved in Eq. (1):

\[
\lambda_0 = -2.074 \text{ W/m} \cdot \text{deg}, \quad \omega = -0.185 \cdot 10^{-2} \text{ 1/deg.}
\]

A relation for calculating the dissociation time taking into account dependence (1) when \( \omega t \leq 0.1 \), but ignoring variations in the porosity of the material with temperature, has been derived by the present authors in [11]. In the present paper no limitations are imposed on the magnitude of \( \omega t \).

The heat being fed to the reacting surface passes through the outer shell of reacted material with a thermal conductivity \( \lambda \). The heat flux at the reacting surface of a spherical particle is determined by solving the stationary thermal-conductivity equation \( \text{div} (\lambda \nabla t) = 0 \) taking into account dependence (1) under the boundary conditions

\[
t = t_0 \quad \text{for} \quad r = r_0, \quad t = t_d \quad \text{for} \quad r = r_1.
\]

The solution to the problem takes the form

\[
t(r) = \frac{1}{\omega} \left[ \sqrt{1 + 2\omega \left( \frac{B - A}{r} \right)^{-1}} - 1 \right],
\]

where

\[
A = \frac{(t_s - t_d) r_g r_1}{r_0 - r_1} + \frac{(t_s - t_d) \omega_r r_1}{2(r_0 - r_1)};
\]

\[
B = t_0 + \frac{\omega_d}{2} + \frac{(t_s - t_d) r_g}{r_0 - r_1} + \frac{(t_g - t_d) \omega_r}{2(r_0 - r_1)}.
\]

The radius of the reacting surface \( r_1 \) is treated hereinafter as variable, i.e., \( r_1 = r \). The heat flux through the reacting surface

\[
q = \frac{\lambda r_s (t_s - t_d)}{r (r_0 - r)} \left[ 1 + \frac{\omega}{2} (t_s - t_d) \right].
\]

The surface temperature of a spherical cone \( t_S \) can be found from the equation of thermal balance:

\[
4 \pi r^2 q = 4 \pi \alpha r^2 \left( t_m - t_d \right).
\]

The temperatures \( t_m \) and \( t_d \) are determined experimentally. The relationship between the temperature difference \( \Delta t_1 = t_s - t_d \) and \( \Delta t_2 = t_m - t_d \), which can be found from Eqs. (7) and (8), is used to obtain the following relation for the heat flux:

\[
q = \frac{\lambda \alpha r_0^2 \Delta t_2}{r [r_0(1 + \omega_d) + \alpha r_0 (r_0 - r)]} \left\{ 1 + \frac{\omega}{2} \left[ 2t_d + \frac{\alpha r_o (r_0 - r) \Delta t_d}{r_0(1 + \omega_d) + \alpha r_0 (r_0 - r)} \right] \right\}. \tag{9}
\]

By representing the dissociation rate by the rate of volumetric change we obtain [5]

\[
q = -Q_{sp} \rho b \frac{c_x}{100} \frac{dr}{dx}. \tag{10}
\]

A comparison of (9) and (10) gives a differential equation for determining the dissociation rate:

\[
\frac{100 \lambda_0 \alpha r_0^2 \Delta t_4}{Q_{sp} \rho b c_x} \frac{dr}{dx} = \frac{2r [r_0(1 + \omega_d) - \alpha r_0] + \alpha r_0^2}{(1 + \omega_d) [r_0(1 + \omega_d) - \alpha r_0] + \alpha r_0^2 + (r_0 - r) \Delta t_d \alpha r_0 \omega}, \tag{11}
\]

and \( r = 0 \) for \( r = r_0 \).

Using the solution to Eq. (11) and the obvious relation

\[
\frac{r}{r_0} = \left( \frac{c_x}{c_0} \right)^{\frac{1}{3}},
\]

for the limiting case \( c_x = 0 \) we obtain a formula for determining the time required for the total dissociation of a single lump:

\[
r_t = \frac{Q_{sp} \rho b r_0}{50 \lambda_0 \alpha \Delta t_4} \left[ K \left[ \frac{1}{3 M} - \frac{N}{2 M^2} + \frac{N^2}{M^3} + \frac{N^3}{M^4} \ln \frac{N}{M + N} \right] + 2K \alpha r_0 \left[ \frac{1}{2 M} - \frac{N}{M^2} - \frac{N^2}{M^3} \ln \frac{N}{M + N} \right] + \alpha^2 r_0^2 \left[ \frac{1}{M} + \frac{N}{M^2} \ln \frac{N}{M + N} \right] \right], \tag{12}
\]

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