The solution is constructed for the problem of thermal slip of a moderately dense gas along a flat surface. The method of half space moments is used.

Thermal slip has been studied in many papers (see [2], for example). As a rule, the Boltzmann equation with a model collision integral in the BGK form [4] has hence been used. The influence of the gas not being ideal on the thermal slip velocity is taken into account here by using the Chapman—Enskog equation for compact gases converted within the scope of the BGK ideas.

Let us consider a gas which is above a wall in temperature gradient field tangential to the wall. Let us introduce a Cartesian coordinate system with origin on the wall surface, $x$ axis along the normal to the wall, and $y$ axis along the wall surface in the direction of $\nabla T$.

The well-known Chapman—Enskog equation for dense gases [1] with a nonlocal collision integral, which is ordinarily expanded in a power series in the small parameter $\sigma/L$ ($\sigma$ is the effective molecular diameter, and $L$ is the characteristic dimension of the problem), with only terms not above the first order in $\sigma/L$ retained:

$$
(v \cdot \nabla) f = \chi \int (f' f_i - f f_i) \sigma^2 g \cdot k dk dv + \chi \int k (f' f_i + f f_i) \sigma^2 g \cdot k dk dv + \frac{1}{2} \int k \cdot \nabla (f' f_i + f f_i) \sigma^2 g \cdot k dk dv,
$$

is the initial equation. Here $g = v_1 - v$ is the relative velocity of the gas molecules; $k$, a vector along the line of centers; $\chi$, a factor taking account of the increase in the collision probability with the rise in gas density. The following expression for $\chi$ can be used for gases of moderate density:

$$
\chi = 1 + \frac{5}{8} b \rho,
$$

where $b = 2/3 \cdot \pi \sigma^3/m$; $\rho = mn$; $n$ is the number of molecules per unit volume and $m$ is the mass of the molecules.

In this case the characteristic dimension is the Knudsen layer thickness which equals the molecule mean free path $\lambda$ in order of magnitude. The ratio $\sigma/\lambda$ is therefore a small parameter. The requirement of smallness of $\sigma/\lambda$ imposes a constraint on the density. Thus, if it is assumed that $\sigma/\lambda \sim 0.1$, then we obtain $n \approx 8.9 \cdot 10^{21}$, which corresponds to a pressure on the order of 300 atm (for hydrogen).

Let us introduce the following notation:

$$
\int dv = n \left( \frac{m}{2 k T} \right)^{3/2} \exp \left[ - \frac{(c - G)^2}{2} \right], \quad c = \left( \frac{m}{2 k T} \right)^{1/2} v, \quad G = \left( \frac{m}{2 k T} \right)^{1/2} u,
$$

$$
n = \int dv, \quad u = \frac{1}{n} \int v dv, \quad \frac{3}{2} k T = \frac{1}{n} \int \frac{m v^2}{2} dv.
$$

Since the influence of the wall on the molecule velocity distribution has a finite radius of action, the distribution function far from the wall should go over into the Chapman—Enskog distribution

$$
f = f^e [1 + \psi(c, y)].
$$

---

where

\[ \psi(c, y) = \frac{1}{n} \left( \frac{2kT}{m} \right)^{\frac{1}{2}} A c y S^{(1)} \frac{\partial}{\partial y} \ln T; \]

\[ A = \frac{1}{\chi} \left( 1 + \frac{3}{5} b p \chi \right) \frac{3n_0}{2kT}; \]

\[ S^{(1)} = \left( \frac{1}{2} \right) - c^2; \eta_0 \text{ is the gas viscosity at the same temperature under normal pressure. The viscosity of a} \]

\[ \text{dense gas is associated with } \eta_0 \text{ by the relation} \]

\[ \eta = \eta_0 \beta p \left( \frac{1}{b p \chi} + \frac{4}{5} + 0.76 b p \chi \right). \]

Near the wall it is necessary to distinguish between the distribution functions of the incident and reflected molecules, which we denote by the superscripts - and +, respectively.

Let us seek the distribution function in the form

\[ f^\pm = f_1^{eq} [1 + \psi(c, y) + \varphi^\pm(c, x)]. \]

Here \( \varphi \) is the correction to the distribution function, which takes care of the influence of the wall. As is shown in [3], \( |\varphi/\partial y| \ll |\varphi/\partial x| \), hence \( \varphi \) can be considered a function of just \( c \) and \( x \).

The main assumption of the BGK method is that the distribution function goes over into a local Maxwell distribution \( f^{eq} \) during one collision, hence, the substitution \( f_1^{eq} \rightarrow f^{eq}, f_1^{eq} \) must be made in all the integrals in the right-hand side of (1). Moreover, the first of the integrals is replaced by the expression \( \nu (f^{eq} - f) \), where \( \nu \) is the collision frequency. Let us also note that \( n \), and therefore \( \chi \), vary slightly within the Knudsen layer limits. Taking the above into account, we substitute (2) into (1) while retaining first-order terms in \( \sigma/\lambda \) here:

\[ (v \cdot \nabla) f^{eq} + (v \cdot \nabla) f^{\pm} = -v f^{eq} (\psi + \varphi) + \int (k \cdot v) \left( \frac{2}{5} - \frac{5}{2} \right) \varphi f^{eq} \frac{\partial f^{eq}}{\partial y} + \]

\[ + \chi \int (k \cdot v) \varphi f^{eq} \frac{\partial f^{eq}}{\partial y} + k \cdot v [n \chi \nu] \frac{\partial f^{eq}}{\partial y} \ln [f^{eq}] + \int (k \cdot v) \varphi f^{eq} \frac{\partial f^{eq}}{\partial y}. \]

It is here taken into account that \( f^{eq} f_1^{eq} = f_1^{eq} f^{eq} \).

The first and third integrals in the right-hand side of (3) are easily evaluated analytically [1]; hence taking into account that the continuity equation and the momentum and energy conservation laws are satisfied far from the wall, we obtain

\[ \frac{m}{kT} \left( 1 + \frac{2}{5} b p \chi \right) v_x v_y \frac{\partial f}{\partial x} + v_y \left( 1 + \frac{3}{5} b p \chi \right) \left( \frac{c^2}{\sigma} - \frac{5}{2} \right) \frac{\partial f}{\partial y} \ln T \]

\[ + v_x \frac{\partial f}{\partial x} = -v (\psi + \varphi) + \chi \int (k \cdot v) \varphi f^{eq} \frac{\partial f^{eq}}{\partial y} \ln [f^{eq}] + \int (k \cdot v) \varphi f^{eq} \frac{\partial f^{eq}}{\partial y}. \]  

We obtain an expression for \( \nu \)

\[ \nu = \frac{2nkT}{3\eta_0} \chi \]

from the condition that \( \psi \) is the Chapman – Enskog correction far from the wall.  

Terms corresponding to the Chapman – Enskog solution

\[ 2 \left( 1 + \frac{2}{5} b p \chi \right) c_x c_y \frac{\partial f}{\partial x} + c_x \frac{\partial f}{\partial x} = -v^* \varphi + \chi \left( \frac{2kT}{m} \right)^{\frac{1}{2}} \int (k \cdot v) \varphi f^{eq} \frac{\partial f^{eq}}{\partial y} \ln [f^{eq}] + \int (k \cdot v) \varphi f^{eq} \frac{\partial f^{eq}}{\partial y}, \]

\[ \nu^* = \left( \frac{m}{2kT} \right)^{\frac{1}{2}} \nu, \quad g^* = \left( \frac{m}{2kT} \right)^{\frac{1}{2}} g \]

vanish in (4) for such a selection of \( \nu \). Let us introduce the new function \( \Phi = 2c_y G + \varphi \), where we seek \( \Phi^\pm \) in the form of a series expansion in Sonine polynomials in velocity space:

\[ \Phi = a_0 c_y + a_1 c_y S^{(1)} + a_2 c_y S^{(2)} \]

\[ = \frac{a_0 + a_0}{2} c_0 + \frac{a_0 - a_0}{2} c_y \text{sign} c_x + \frac{a_1 + a_1}{2} c_y S^{(1)} + \frac{a_2 + a_2}{2} c_y S^{(2)} \text{sign} c_x, \]

\[ \frac{a_0 + a_0}{2} c_0 + \frac{a_0 - a_0}{2} c_y \text{sign} c_x + \frac{a_1 + a_1}{2} c_y S^{(1)} + \frac{a_2 + a_2}{2} c_y S^{(2)} \text{sign} c_x, \]

934