OPTIMAL HEATING SURFACE IN A MULTISTAGE COUNTERCURRENT FLUIDIZED-BED HEAT EXCHANGER

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Equations are derived for calculating the intermediate temperatures of the heating medium such as to give the minimal total surface area in a multistage countercurrent heat exchanger having tubes immersed in a fluidized bed.

Heat exchangers having surfaces immersed in fluidized beds are increasingly being used in industry, on account of the properties of fluidized beds such as high heat-transfer coefficient, isothermal conditions in the bed, and absence of local overheating.

On the other hand, the isothermal condition is a disadvantage, since it restricts the maximum temperature of the cooling medium. The heat-balance equation for a one-stage exchanger is as follows (here and subsequently it is assumed that the heating medium passes within the tubes, while the cooler medium passes through the bed):

\[ w(t_i - t_2) = \theta_i - \theta_s, \]  
(1)

which goes with the condition for the temperature difference between the heating fluid and the cooling one:

\[ t_2 \gg \theta_i, \]  
(2)

which gives an expression for the temperature of the cooler fluid in a one-stage equipment; in the limiting case, for \( t_2 = \theta_i \), the maximum temperature in the cooler medium is

\[ \theta_{1\text{max}} = \frac{\theta_i + \theta_s}{1 + w}. \]  
(3)

One therefore usually employs a multistage system in order to raise the final temperature of the cooler medium.

The gas temperature falls in a single stage of a fluidized-bed heat exchanger, while the temperature of the bed remains constant, and therefore one cannot say one has countercurrent flow or direct flow in a single stage; however, in a multistage system it is possible to distinguish the two types.

In a direct-flow system, the two fluids enter the first stage and pass in series through all stages to emerge from stage n. In the countercurrent case, the two fluids enter at opposite ends of the chain and move in opposite directions.
Here we consider a countercurrent system as shown in Fig. 1.

The initial data for the design are \( w_h, w_c, t_1, \theta_1, \theta_{n+1} \); the calculation has to provide the areas \( F_i \) and \( F \) of the surfaces. The calculation is based on the heat-balance equation (4) and the heat-transfer equation (5) for each of the individual stages:

\[
\begin{align*}
\omega_h(t_i - t_{i-1}) &= \omega_c(\theta_i - \theta_{i+1}), \quad \text{(4)} \\
\omega_h(t_i - t_{i-1}) &= \frac{K_i F_i (t_i - t_{i+1})}{\ln \frac{t_i - \theta_i}{t_{i+1} - \theta_i}}, \quad \text{(5)}
\end{align*}
\]

From (5) we get \( F_i \) and \( F \):

\[
F_i = \frac{\omega_h}{K_i} \ln \frac{t_i - \theta_i}{t_{i+1} - \theta_i}. \quad \text{(6)}
\]

The total surface area of all heat exchangers is

\[
F = \sum F_i = w_h \left[ \frac{1}{K_1} \ln \frac{t_1 - \theta_1}{t_2 - \theta_1} + \cdots + \frac{1}{K_n} \ln \frac{t_n - \theta_n}{t_{n+1} - \theta_n} \right]. \quad \text{(7)}
\]

The areas in the preceding exchangers (along the path of the hotter medium) decrease as the intermediate temperatures \( t_2, t_3, \ldots, t_n \) increase, while the areas in the later stages increase. There is a turning point in the sum of the areas in this system of fluidized-bed exchangers having intermediate temperatures \( \Delta F_1 = \Delta F_{i+1} \), which can be represented as \( F = f(t_2, t_3, \ldots, t_n) \), the minimum occurring for \( w_h, w_c, t_1, t_{n+1}, \theta_1, \theta_{n+1} \), all constant.

To determine the critical points in (7) we take the partial derivative

\[
\frac{\partial F}{\partial t_i} = w_h \left[ \frac{K_{i-1}(t_{i-1} - \theta_{i+1})(t_i - \theta_{i-1}) - K_i(t_i - \theta_i)(t_{i+1} - \theta_i)}{K_{i-1}K_i(t_i - \theta_i)(t_{i+1} - \theta_i)(t_i - \theta_{i-1})} \right]. \quad \text{(8)}
\]

The partial derivative of (8) is zero, infinite, or has no value at a critical point. We equate the numerator and denominator in (8), in turn, to zero to get for heat exchanger \( i \) that

\[
K_{i-1}(t_{i+1} - \theta_{i+1})(t_i - \theta_{i-1}) - K_i(t_i - \theta_i)(t_{i+1} - \theta_i) = 0, \quad \text{(9)}
\]

\[
t_i - \theta_i = 0, \quad \text{(10)}
\]

\[
t_{i+1} - \theta_{i+1} = 0, \quad \text{(11)}
\]

\[
t_i - \theta_{i-1} = 0. \quad \text{(12)}
\]

It is found that (7) has a minimum for the \( t_i \) defined by (9).

We have from (4) that

\[
\omega t_i - \theta_i = c_1 - \text{const}. \quad \text{(13)}
\]

We put

\[
t_{i+1} - \theta_{i+1} = c_2. \quad \text{(14)}
\]

We solve (9) with (13) and (14) to get

\[
t_i^2K_i\omega(1 - w) + t_i[c_2(K_i - K_{i-1}) + 2wK_i c_{i+1}] - K_{i-1}\theta_{i-1}c_2 - K_i c_1(c_1 - t_{i-1}) = 0. \quad \text{(15)}
\]