DETERMINATION OF THE TEMPERATURE AND DENSITY OF A HEAT FLUX FROM THE SOLUTION OF THE INVERSE HEAT CONDUCTION PROBLEM FOR A THERMALLY DESTRUCTIBLE MATERIAL


The problem of determination of a heat flux density from the state of a material after heating is considered.

When construction elements and heat-protective coatings, consisting of polymer composites, are subjected to heating, they undergo thermal destruction, and their state after cessation of heating provides some information about the character of such heat action.

In the present work we consider how this information may be used to reconstruct the temperature dependence \( T(t) \) at some point of a structure or to determine the specific heat flux \( q(t) \) acting on a boundary. This may be done if there is a mathematical model of the thermal destruction process (the irreversible change of internal parameters of the material, viz., the density, the concentration of the constituents of the material, etc.). Such an approach is promising since it makes it possible to obtain a greater amount of information in full-scale tests by using of structural and heat-protective materials as heat indicators.

Below we illustrate use of information obtained from thermogravimetric analysis of material samples before and after heat loading.

We consider the simplest phenomenological model of a thermally destructible material [1]. According to [1], for thermogravimetric analysis of some thermally destructible materials the destruction process may be approximated by a sum of several reactions (stages) with different kinetic parameters. For the case of parallel independent stages the system of equations is as follows:

\[
dG_i/dt = -A_i G_0 C_i^n \exp \left(-E_i/T(t)\right), \quad i = 1 \ldots NST. \tag{1}
\]

Prior to thermal loading \( C_i = 1 \), and after cooling

\[
C_i = \exp \left(-\int_0^t \exp \left(\ln A_i - E_i/T(t)\right) dt\right); \quad i = 1 \ldots NST. \tag{2}
\]

The problem is subdivided into three problems for determination of: 1) \( A_i, E_i, C_{0i} \) for the initial material (formulation of the mathematical model of material destruction); 2) \( C_i \) for the material after heating; 3) \( T(t) \) in terms of \( C_i \).

Problem 1 is solved analogously to [1, 2] (by successive subtraction of stages); problem 2 by optimization methods (for the given kinetic parameters the \( C_i \) values are chosen so as to provide the best approximation of the thermogravimetric curve of the material after heating).

To solve problems 1 and 2, we have composed a set of programs in Basic for an IBM PC AT and verified it for several materials. Subsequently, we analyze only problem 3, considering that we already have a model of the material (the values of the kinetic parameters) and the \( C_i \) values.

The system of equations for the concentrations of the stages after heating \( T(t) \) has the form \((-\ln (C_i)/A_i = CK_i(tk))\):

\[ CK_i(tk) = \int_0^{tk} \exp\left(-\frac{E_i}{T(t)}\right) dt, \quad i = 1 \ldots NST. \]  

(3)

We must find \( T(t) \) by using the known \( CK_i \), i.e., solve a nonlinear inverse problem (reconstruct the cause from its effect).

System of equations (3) has infinitely many solutions because the interchange of two sections of the curve does not change the value of the integral.

In order to single out the necessary solution we will use a priori information that contains the time interval \( tk \) (in order to eliminate the zeroth values) and requirements on smoothness and symmetry of the solution (to avoid the rearrangement symmetry). Knowing a sufficiently exact initial approximation \( T(t) \), the problem may be linearized to give the system

\[ CK_i(t) = \int_0^t K_i(T(\tau)) \Delta T(\tau) d\tau, \quad i = 1 \ldots NST, \]  

(4)

which is the discrete analog of the Fredholm equation of the first kind. Problems that are close in character arise in determining by satellite the temperature, moisture, and concentration of gases of the Earth or planets by their radiation spectrum [3-6].

If the change in the temperature \( T(t) \) is not arbitrary but is determined by some heat transfer and heat conduction problems, then some of the solutions of system of equations (3) may be realized only for nonphysical values of the heat flux, thus permitting regularization of problem 3 by solving the inverse heat conduction problem. As a result, we arrive at a problem that differs from those described in [7, 8] and is close to problems of optimal control.

We now consider a thin plate made of a material that is destroyed on heating according to some known law (this is the kinetic equation of a first-order reaction):

\[ C(T) \frac{dT}{dt} = q(t) - \varepsilon aT^2(t), \]  

(5)

\[ G(t) = G_0 \exp\left(-A \int_0^t \exp\left(-\frac{E}{T(t)}\right) dt\right), \]  

(6)

\[ t \in [0, tk]; \quad T(0) = T_0. \]

We assume that \( G(tk) = GK \) is known from experiment, and it is necessary to determine \( q(t) \). We will seek the heat flux in the form \( q(t) = \overline{q} q_0(t) \), where \( q_0(t) \) is the a function given a priori; \( \overline{q} \) is a parameter (a relative heat flux). We seek a \( \overline{q} \) that minimizes the discrepancy \( (GK - G(tk, \overline{q}))^2 \).

If the rather natural assumption that with an increase in the heat flux and the temperature the final mass of the material decreases \( (dG(tk, \overline{q})/d\overline{q} < 0) \) is fulfilled, then a solution exists and it is unique. Thus, a thermally destructible material may be used for determination of the relative heat flux similarly to heat-sensitive paints or irradiated crystals [9].

We now consider a material whose thermal destruction is described by several parallel independent reactions:

\[ G_i(t) = G_{0i} \exp\left(-A_i \int_0^t \exp\left(-\frac{E_i}{T(t)}\right) dt\right), \quad i = 1 \ldots NST, \]  

(7)

\[ G(t) = \Sigma G_i(t) \quad \text{or} \quad G(t) = \text{const}. \]