GENERALIZED ISOTHERMAL COUETTE FLOW OF A NON-NEWTONIAN LIQUID IN A SLOWLY CONVERGING CHANNEL WITH COMPLEX SHEAR

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There is an analysis of the flow of an anomalous-viscosity liquid with a power-law rheological equation in the converging screw channel of a screw pump (an extruder, a mixer, etc.). Circulation of the liquid in the channel is taken into account.

The model usually adopted for the flow of a liquid in the converging screw channel of a screw machine is simple shear flow between two infinite plates inclined at some angle with respect to each other. A pressure gradient acts in the region between the plates. This problem was treated in [1, 2] for a Newtonian liquid, and that for a non-Newtonian liquid, obeying a power-law rheological equation, was treated in [3-5]. Since this model is valid only for screws with a small pitch angle and for liquids whose properties are not greatly different from those of a Newtonian liquid, we feel it is worthwhile to treat the analogous problem for the case of a complex shear flow.

We consider the flow of a liquid with a power-law rheological equation in the slowly converging screw channel of a screw pump. The screw is shown in Fig. 1a. We assume that there are no gaps between the thread of the screw and the housing, and we assume that the initial depth H of the channel is less than its width S and much less than the screw radius. Under these assumptions we can treat the liquid flow within the channel as flow between infinite plates (Fig. 1b). The lower plate is fixed, while the upper plate moves at a velocity \( v_0 \). The x axis is along the channel, the y axis is along its width, and the z axis is along its depth. We neglect the inclination of the plates in the y direction. Acting along the x and y axes are pressure gradients \( \frac{\partial p}{\partial x} = A_1 \) and \( \frac{\partial p}{\partial y} = A_2 \). We assume that the liquid flow rate \( Q \) is given.

A qualitative examination of the flow pattern in a slowly converging channel shows that, by virtue of the continuity condition, the velocity curves and thus the pressure gradients \( A_1 \) and \( A_2 \) vary along the length of the channel. At certain values of the parameters in the gap between the plates there can exist a cross section \( h_x = h_x \), in which we have \( A_1 = 0 \). In this case, in the region \( H > h_x > h_0 \), the pressure gradient \( A_1 > 0 \) reduces the flow rate of the product, while in the region \( h_0 < h_x < h \), the gradient \( A_1 < 0 \) increases the flow rate. If \( A_1 \) changes sign in the gap between the plates, the pressure initially increases from the entrance toward the exit and then decreases; if, on the other hand, \( A_1 \) changes sign beyond the channel \( h_* \) the pressure increases continuously from the entrance of the channel to the exit.

We replace the inclined plane by a series of steps consisting of plane regions of length \( dx \) parallel to the \( x \) axis, each shifted by an amount \( dz \) with respect to the adjacent region. Within each such step the decrease in the channel depth leads to the appearance of a pressure drop \( dp \) in this region. We assume that within each step the liquid flows as it would between parallel plates. We neglect the velocity component \( w_2 \).

As the rheological equation we adopt a power law, which can be written in the following manner for this type of flow [6]:

\[
\tau = B \left( \frac{I_2}{2} \right)^{\frac{1}{2}} - \Delta,
\]

where

\[
\frac{I_2}{2} = \left( \frac{\partial \omega_x}{\partial z} \right)^2 + \left( \frac{\partial \omega_y}{\partial z} \right)^2.
\]

The equations of motion along the x and y axes are

\[
\frac{\partial \tau_{xx}}{\partial z} = \pm A_1, \quad \frac{\partial \tau_{yy}}{\partial z} = A_2,
\]

and their solution is

\[
\tau_{xx} = \pm A_1 (z - c_1 h_o), \quad \tau_{yy} = A_2 (z - c_2 h_o).
\]

Using the condition that the liquid adheres to the plates and the condition that the flow rate in the direction transverse to the channel axis vanishes, we can write the joint solution of Eqs. (1) and (3) in the following form* [7]:

\[
u_x = \frac{w_x}{V_0} = \pm \frac{1}{\alpha} \int_0^h f(\xi, c_1, c_2, a)(\xi - c_1) \, d\xi,
\]

\[
u_y = \frac{w_y}{V_0} = \frac{1}{\alpha} \int_0^h f(\xi, c_1, c_2, a)(\xi - c_2) \, d\xi,*
\]

*In Eqs. (4)–(6) and below the plus sign corresponds to flow with \( A_1 \geq 0 \), and the minus sign corresponds to flow with \( A_1 \leq 0 \).