MASS EXCHANGE BETWEEN A SOLID SPHERICAL BODY 
AND A CURRENT-CARRYING LIQUID

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The problem of the determination of the action of an electric current on mass exchange in a solid body - liquid system is solved.

It is known that near a solid body immersed in an electrically conducting liquid through which an electric current flows, the liquid will move. The cause of this motion is the action of the magnetic field of the electric current on the current-carrying liquid. Chow [1] determined the velocity field about a spherical particle under conditions of superposition of two slow motions: the forced motion of the particle and the motion of the liquid caused by the electric current. The stream function in this case has the form

$$\psi = \frac{Ua^2}{2} \left[ \left( \frac{r}{a} \right)^2 - \frac{3}{2} \left( \frac{r}{a} \right) + \frac{1}{2} \left( \frac{a}{r} \right) \right] \sin^2 \theta - K \left[ \left( \frac{r}{a} \right)^2 - \frac{5}{2} \left( \frac{a}{r} \right) + \frac{1}{2} \left( \frac{a}{r} \right)^2 \right] \sin^2 \theta \cos \theta,$$  \hspace{1cm} (1)

where $K = \mu_0 I_0 a^3 / 8 \pi \sigma_0$.

Equation (1) with sufficient accuracy is valid when the numbers $Re << 1$ and $Re_m << 1$. The solution of the formulated hydrodynamic problem established prerequisites for the solution of a problem of mass exchange under the same conditions.

We consider the steady-state transport of matter from the surface of a dissolving sphere into the surrounding liquid. Assuming that the case being considered corresponds to large values of the criterion $Pe$ (which is caused by values of the criterion $Pr = 10^3 - 10^6$) we will assume that the representations concerning the diffusion layer that arises on the surface of the sphere are correct. The system of differential equations and boundary conditions that determines the concentration of solute $c = c(r, \theta)$ has the form

$$u_r \frac{\partial c}{\partial r} + u_\theta \frac{\partial c}{\partial \theta} = D \frac{\partial^2 c}{\partial \rho^2},$$

$$c(a, \theta) = c_a, \; c(\infty, \theta) = c_\infty, \; \frac{\partial c}{\partial \theta} \bigg|_{\rho=a} = 0.$$  \hspace{1cm} (2)

System (2) with the use of Eq. (1) is solved in Mises' variables and leads to the result

$$\frac{c_a - c}{c_a - c_0} = \frac{1}{1.16} \int_0^z \exp \left( - \frac{4}{9} x^3 \right) dx,$$

where

$$z = 0.397 \frac{(r-a)}{a} \frac{Pe^{1/3}}{\sqrt[3]{\frac{4}{3}} \int_0^z \sqrt{7 K \cos \theta + 3 \sin^2 \theta} dx \sqrt[3]{\frac{4}{3}} \cos x + 3 \sin x dx}.$$

The sign will be positive if the directions of motion in the region being considered coincide.

For $K > 3/7$ the mass flow from the entire surface of the sphere per unit time equals

$$J = 2na 0.342D(c_0 - c_1) Pe^{1/3} \left[ \frac{\pi - \arcsin \frac{3}{7K}}{\arcsin \frac{3}{7K}} \int_0^1 \sqrt{7K \cos \theta - 3 \sin^2 \theta} \theta \, d\theta \right]$$

$$\approx 2 \pi a 0.342 D \cdot \frac{3}{7K} \left( \frac{7K \cos \theta - 3 \sin^2 \theta}{\sqrt{7K \cos \theta - 3 \sin^2 \theta}} \right)^{1/3} \int_0^1 \sqrt{7K \cos \theta - 3 \sin^2 \theta} \, d\theta$$

Hence,

$$Nu = 0.4825 Pe^{1/3} \left[ \frac{\pi}{\arcsin \frac{3}{7K}} \left( \frac{7K \cos \theta - 3 \sin^2 \theta}{\sqrt{7K \cos \theta - 3 \sin^2 \theta}} \right)^{1/3} \right]$$

For $K > 3/7$, then

$$Nu = 0.4825 Pe^{1/3} \left[ \frac{\pi}{\arcsin \frac{3}{7K}} \left( \frac{7K \cos \theta - 3 \sin^2 \theta}{\sqrt{7K \cos \theta - 3 \sin^2 \theta}} \right)^{1/3} \right].$$

where $p_1^2 = 14K / (7K + 3)$; $p_2^2 = 14K / (7K - 3)$. For large $K$ with accuracy to $O(1 / K)$ from (3) we have

$$Nu = 1.2 Pe^{1/3} K^{1/3}.$$