A method is proposed for calculating the thermal resistance of a system of parallelepipeds with a local source which is encountered in the analysis of the thermal conditions in hybrid integral microcircuits.

Many applied problems, particularly problems associated with the analysis of thermal conditions in microelectronic devices, reduce to a thermal model which is a pyramid of n unlike parallelepipeds of different sizes (Fig. 1a). In the actual construction, the parallelepipeds forming the pyramid are the backing, "chips," adhesive layer, switching plate, base, etc. [1]. There are rectangular, flat energy sources on the upper surface of the first parallelepiped. Thermal contact between adjacent surfaces is assumed ideal. Heat dissipation from the lower surface of the n-th parallelepiped obeys Newton's law and is characterized by a heat-exchange coefficient α; there is no heat exchange at the lower surfaces.

The exact mathematical description of the temperature field in such a system is rather complex and hardly can be used for practical purposes. Calculation of the thermal resistance from the source to the environment is usually based on the construction of an equivalent circuit representing a chain of series-connected thermal resistances [1-4]. It is further assumed that the interfaces are isothermal.

We analyzed the possibility of such an approach for the following problem: a bounded cylinder with a local energy source on one end and boundary conditions of the first and third kind on the opposite end. The thermal resistance from the source to the environment was calculated in the two cases. A comparison of the results showed that the values of the thermal resistance can differ by almost a factor of two for given values of the Biot number and given ratios of cylinder and source sizes. Therefore, determination of the thermal resistance of this system must be carried out with consideration of heat-transfer conditions at the heat-releasing surfaces of each body. A method is proposed below for which sequential application provides an accuracy sufficient for practical purposes without significant complication of the computational process.

**Method for Determination of Thermal Resistance**

We shall show that for this system of bodies, the problem of determining the total thermal resistance from energy source to environment can be reduced to a problem of determining the thermal resistance of the first parallelepiped, the heat-transfer conditions at the lower boundary of which are characterized by an equivalent coefficient $\alpha_1$ that includes the effect of all the other parallelepipeds (Fig. 1c).

To determine the value of $\alpha_1$, we consider successively the temperature field of each i-th parallelepiped, for which the thermal model can be represented in the following manner: on the upper face of the parallelepiped, there is a flat energy source with an intensity $p$ and area $S_{i-1}$; on the lower surface, heat transfer is characterized by a heat-exchange coefficient $\alpha_i$, which takes into account the effect of all the remaining $(n-1)$
parallelepips with the other surfaces thermally insulated (Fig. 1b).

For the lowest parallelepiped \( i = n \), the quantity \( \alpha_n \) is known and characterizes the actual conditions of heat exchange with the environment, \( \alpha_n = \alpha \). We define the thermal resistance of the \( n \)-th parallelepiped from its contact surface to the environment in the following manner:

\[
R_i = \frac{h_i}{\gamma_i} \alpha_i
\]

(1)

Here \( \gamma_i \) is a coefficient which takes into account spreading of the thermal flux over the thickness of the parallelepiped.

To determine \( \alpha_{n-1} \), the equivalent coefficient of heat exchange for the \((n - 1)\)-th parallelepiped, we represent the quantity \( R_n \) in the form

\[
R_n = \frac{1}{\alpha_n S_n}.
\]

(2)

We then have from Eqs. (1) and (2)

\[
\alpha_{n-1} = \frac{h_n \gamma_n (\alpha_n, h_n, S_n, S_{n-1})}{\alpha_n S_n}.
\]

(3)

Now considering the \((n - 1)\)-th parallelepiped, we express its thermal resistance similarly through the corresponding geometric parameters, coefficient of thermal conductivity \( \lambda_{n-1} \), and coefficient of heat exchange \( \alpha_{n-1} \). Repeating this operation successively for each \( i \)-th parallelepiped, we obtain

\[
\alpha_i = \frac{h_{i+1}}{\lambda_i S_{i+1} \gamma_{i+1} (\alpha_{i+1}, h_{i+1}, S_{i+1}, S_i)},
\]

(4)

\[
R_i = \frac{h_i}{\lambda_i S_i \gamma_i (\alpha_i, h_i, S_i, S_{i-1})}.
\]

(5)

In the final analysis, we obtain the equivalent heat-exchange coefficient \( \alpha_1 \) at the boundary between the first and second parallelepipeds (Fig. 1c). Then the thermal resistance from the source on the upper surface of the first parallelepiped to the environment has the form

\[
R_1 = \frac{h_1}{\lambda_1 S_1 \gamma_1 (\alpha_1, h_1, S_1, S_2)}.
\]

(6)

Analytic expressions for the determination of the quantities \( \gamma_1 \) can be obtained by both exact [5] and approximate [6, 7] methods. Furthermore, it is usually assumed that the thermal flux density in the region occupied by the source is uniformly distributed, i.e., \( q = \text{const.} \)

In this problem, the assumption \( q = \text{const.} \) can be considered valid only for the top parallelepiped, on the surface of which the actual sources are located. For the remaining parallelepipeds, the thermal flux density...