GENERAL PROBLEMS OF HEAT EXCHANGE
IN A MOVING LAYER

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An approximate analytical solution is obtained for the problem of steady heat exchange in a moving layer in the presence of heat and mass sources in the gas stream. A numerical-analytical method is developed for the solution of the problem of nonsteady heat exchange of a layer by convection or radiation with the simultaneous action of different disturbing factors.

Heat exchange in a layer moving with variable velocity represents a general case of layer heat exchange, since a stationary layer can be considered as moving with zero velocity. A method of solving problems of heat exchange in a stationary layer based on the use of a general solution of the equation of thermal conduction was examined in [1]. This method is applied below to problems of heat exchange in a moving layer formulated in general form.

First let us examine the steady mode of heat exchange between a layer of massive bodies of the simplest shape and an opposing gas stream in which sources of mass and heat act.

The effect of mass sources is expressed in variation in the flow rate of gas along the length of the layer. The flow rate of the gas and the power of the heat sources are given in the form of arbitrary functions of the time the body stays in the oven or of its coordinate relative to the entrance to the oven. The heat losses are proportional to the average temperature of the gas in the oven. We neglect heat conduction along the layer.

The initial system of equations and the boundary conditions have the following form:

- equation of heat conduction
  \[
  \frac{\partial t(r, Fo)}{\partial Fo} = \frac{\partial^2 t(r, Fo)}{\partial r^2} + \frac{v}{r} \frac{\partial t(r, Fo)}{\partial r},
  \]
- equation of thermal balance of the gas stream
  \[
  \tilde{T}(Fo) = \gamma(Fo) [T(Fo) - 1] + Q(Fo) - \chi' Fo,
  \]
- boundary conditions
  \[
  \frac{\partial t}{\partial r}
  \begin{cases}
  |_{r=1} = Bi[T(Fo) - t(1, Fo)]; & \frac{\partial t}{\partial r}
  \begin{cases}
  \bigg|_{r=0} = 0; \\
  Fo = 0, t = t^a(r), T = 1;
  \end{cases}
  \\
  t = \frac{t^a - t_0}{t_0 - t_s}; T = \frac{t^a - t_s}{t_0 - t_s}; Fo = \frac{\alpha r}{R^2}; Bi = \frac{\alpha R}{\lambda};
  \end{cases}
  \]
  \[
  r = \frac{y}{R}; \gamma(Fo) = \frac{V_g(Fo)c_g}{V_1c_s}; Q(Fo) = \frac{q(Fo)}{V_1c_s(t_0 - t_s)};
  \]
  \[
  \chi' = \frac{2\pi\lambda}{2(t_0^3 - t_s^3)} \int_{R^2} R^4 dp.
  \]


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The general solution of Eq. (1) with the condition that the gas temperature is some function of the time is well known [2, 3]

\[ t = T - \sum_{n=1}^{\infty} A(\mu_n, r) \exp(-\mu_n^2 Fo) \left[ 1 + \int_{\delta}^{Fo} \frac{dT}{d\omega} \exp(\mu_n^2 \omega) d\omega \right] \]

\[ + \sum_{n=1}^{\infty} A'(\mu_n, r, t^0) \exp(-\mu_n^2 Fo). \]  

(5)

Since in the majority of cases the processes of heating of the substance in the moving layer are completed when \( Fo > 0.5 \), in the solution of (5) we confine ourselves to the first terms of the sums. An expression for the average-mass temperature of the body follows from (5):

\[ t = T - B(\mu) \exp(-\mu^2 Fo) \left[ 1 + \int_{\delta}^{Fo} \frac{dT}{d\omega} \exp(\mu^2 \omega) d\omega \right] \]

\[ - B'(\mu, t^0) \exp(-\mu^2 Fo). \]  

(6)

The root \( \mu \) of the characteristic equation and the coefficients \( A, A', B, \) and \( B' \) are known from the solution of the equation of thermal conduction for a single body with boundary conditions of the third kind and are presented in the literature [2].

Let us differentiate (6) with respect to \( Fo \):

\[ \frac{dt}{dFo} = \frac{dT}{dFo} + \mu^2 B \exp(-\mu^2 Fo) \left[ 1 + \int_{\delta}^{Fo} \frac{dT}{d\omega} \exp(\mu^2 \omega) d\omega \right] \]

\[ - B' \exp(-\mu^2 Fo) \]  

(7)

Using (6) we transform Eq. (7) to the form

\[ \frac{dt}{dFo} = \frac{dT}{dFo} + \mu^2 (T - \bar{t}) - B \frac{dT}{dFo}. \]  

(8)

Based on the fact that Eq. (8) must satisfy the condition (2), we arrive at a differential equation relative to the gas temperature

\[ \frac{dT}{dFo} + M(Fo) \frac{dT}{dFo} = N(Fo), \]  

(9)

for which the general integral has the form

\[ T = \exp \left[ - \frac{\delta}{\gamma M dFo} \right] \left[ 1 + \int_{\delta}^{Fo} N \exp \left[ \frac{\delta}{\gamma M dFo} \right] dFo \right], \]  

(10)

\[ M = \frac{d\gamma}{dFo} - \frac{\mu^2 + \gamma \mu^2}{\gamma - 1 + B}; \quad N = \frac{d\gamma}{dFo} - \frac{dQ}{dFo} + \gamma \mu^2 - Qu^2 + x' + x' \mu^2 Fo}{\gamma - 1 + B}. \]

The average-mass temperature of the body is determined from the condition (2) while the temperature at any point through the thickness can be found from the following equation, obtained through substitution from (6) into (5):

\[ t(r, Fo) = T + \frac{A}{B} (T - \bar{t}) - \frac{AB'}{B} \exp(-\mu^2 Fo). \]  

(11)

If \( t^0 = \text{const} \) in the condition (4) then according to [2] we have \( A' = At^0, B' = Bt^0, \) and the last term in (11) is reduced to zero.

Let us examine a particular case of the solution (10) which is characteristic for continuous ovens with multizone heating, in which the sources of mass and heat are concentrated at the junctions of the zones and can be expressed by step functions: