LIMITING VISCOSITY OF FERROMAGNETIC SUSPENSIONS IN A STRONG MAGNETIC FIELD

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The effective viscosity of a suspension of ferromagnetic particles with anisotropy of the "easy-axis" type is calculated. The case considered is that in which the magnetic field in which the suspension is flowing greatly exceeds the internal anisotropy field, and the concept of the "frozen" magnetic moments of the particles is inapplicable. The relationship between the viscosity and the anisotropy field is established. The question as to the magnitude of the viscosity for an arbitrary ratio of the internal and external fields is discussed.

1. The concept of rotational viscosity developed in [1] enables us to explain the experimentally observed [2, 3] increase in the viscosity of suspensions of ferromagnetic particles under the influence of an external magnetic field. The reason for this effect is as follows. In the absence of a field the angular velocity of rotation of the particles suspended in the flow $\Omega = \frac{1}{2} \text{rot } v$ is equal to the local angular velocity of rotation of the liquid $\omega = \frac{1}{2} \text{rot } v$. The viscosity is in this case described by the Einstein equation

$$\eta = \eta_0 \left(1 + \frac{5}{2} \psi \right)$$

(1,1)

where $\eta_0$ is the viscosity of the carrier liquid, $\psi$ is the volumetric concentration of the solid phase. On placing the suspension in a uniform magnetic field $H$ the latter exerts an orientation effect on the magnetic moments of the particles $\mu$, preventing the free rotation of the particles in the vortical flow. The difference in angular velocities so arising $\Omega - \omega$ corresponds to the moment of the frictional forces $6\eta_0 V(\Omega - \omega)$, where $V$ is the volume of one spherical particle. This additional internal friction manifests itself as an increase in the effective viscosity of the suspension: $\eta_e = \eta + \eta_r$, where $\eta_r$ is the rotational viscosity.

The quantity $\eta_r$ depends considerably on the magnetocrystalline-anisotropy energy of the ferromagnetic material, which determines the interaction between the magnetic moment $\mu$ and the rotational degrees of freedom of the particle. In the absence of such an interaction (model of "free" magnetic dipoles) the orientations of the vector $\mu$ in the direction of the field $H$ does not interfere with the free rotation of the particle in the flow ($\omega = \Omega$), so that in this model $\eta_r = 0$. In the limiting case of strong interaction, when the magnetic moment is rigidly connected to the body of the particle (model of "frozen" dipoles), an applied field greatly impedes its rotation, and for a sufficiently strong field the rotation of the particle in the flow stops completely, being replaced by sliding ($\omega = 0$) along the corresponding shear plane. The rotational viscosity $\eta_r(H)$ then reaches its limiting value (saturation effect) equal to [4]

$$\eta_r(\infty) = \frac{5}{2} \eta_0 \psi$$

In real ferromagnetic crystals a type of relationship intermediate between the models of free and frozen dipoles is established between the direction of the vector $\mu$ and the crystallographic axes of the particle: For a finite magnetic-anisotropy energy we may only speak of a partial freezing of the magnetic moment. However strong the external field, even if it maintains the orientation of the moments $\mu$ absolutely constant, it cannot completely prevent the rotation of the particles due to hydrodynamic forces.

The limiting value of the viscosity $\eta_r(\infty)$ of any real magnetic suspension should lie in the range

$$0 < \eta_r(\infty) < \frac{5}{2} \eta_0 \psi$$

We shall now calculate $\eta_T(\infty)$ for a suspension of ferromagnetic particles with anisotropy of the "easy-axis" type.

2. In stable magnetic colloids ferromagnetic particles with linear dimensions $\mu \sim 10^{-5}$-$10^{-6}$ cm are generally employed. For the dimensions indicated each particle forms an individual uniformly magnetized domain with a dipole moment $\mu = M_S V$, where $M_S$ is the saturated magnetization of the particle material. The energy of such a particle in an external field is determined by the equation

$$ U = -\mu H (\mathbf{e} \cdot \mathbf{h}) - KV (\mathbf{e} \cdot \mathbf{n})^2, \quad \mathbf{e} = \mathbf{\mu} / \mu, \quad \mathbf{h} = \mathbf{H} / H $$

(2.1)

where $K$ is the energy density of magnetocrystalline anisotropy, $\mathbf{n}$ is the unit vector in the direction of the axis of easy magnetization.

Every deviation of the vector $\mathbf{e}$ from the equilibrium orientation defined by the direction of the effective field

$$ \mathbf{H}_e = -\mathbf{\mu}^{-1} \frac{\partial U}{\partial \mathbf{e}} = H \mathbf{h} + 2K M_S^2 \mathbf{n} (\mathbf{e} \cdot \mathbf{n}) $$

(2.2)

is accompanied by a Larmor precession of the magnetic moment $\mu$ around $\mathbf{H}_e$. It should be remembered that the Larmor-precession attenuation time ($\lesssim 10^{-5}$ sec) is small compared with any hydrodynamic times; hence at every instant of time the internal state of the particle may be regarded as equilibrium ($\mathbf{e} \| \mathbf{H}_e$).

We see from (2.2) that the magnitude and direction of $\mathbf{H}_e$ are determined by the vector sum of the external field $\mathbf{H}$ and the anisotropy field $H_A = 2K / M_S$. Hence for $H_A \gg H$ it is permissible to assume a frozen-in state of the moment ($\mathbf{e} \| \mathbf{n}$), as in [1, 5]. Omitting the inessential constant in the expression for $U$ we obtain $U = -\mu H (\mathbf{e} \cdot \mathbf{n})$ from (2.1). The moment of the magnetic forces acting on the particle

$$ \mathbf{m} = -(\mathbf{n} \times \partial U / \partial \mathbf{n}) $$

(2.3)

is then equal to $(\mathbf{\mu} \times \mathbf{H})$.

For $K = 0$ ($H_A = 0$) we have an "isotropic" magnetic crystals in which the relationship between the magnetic and the mechanical degrees of freedom is broken (model of free dipoles). In this case the rotating moment $\mathbf{m} = 0$, since the magnetic energy $U = -\mu H (\mathbf{e} \cdot \mathbf{n})$ does not depend on the orientation $\mathbf{n}$ of the easy axis of the particle.

In order to determine the limiting viscosity of a suspension of ferromagnetic particles with finite anisotropy in a strong external field we must consider the case in which

$$ H \gg H_A, \quad H_A = 2K / M_S $$

(2.4)

It follows from the condition of equilibrium $\mathbf{e} \| \mathbf{H}_e$ and Eq. (2.2) for the effective field $\mathbf{H}_e$ that on satisfying inequality (2.4) the magnetic moment of the particle may be regarded as parallel to the applied field: $\mathbf{e} = \mathbf{h}$. For the magnetic energy of the particle (2.1) and the rotating moment (torque) (2.3) we than obtain

$$ U = -KV (\mathbf{n} \cdot \mathbf{h})^2, \quad \mathbf{m} = 2KV (\mathbf{n} \cdot \mathbf{h}) (\mathbf{n} \times \mathbf{h}) $$

(2.5)

Subsequently instead of the pseudovectors we shall frequently use their dual antisymmetric tensors; for example, for (2.5) we have

$$ m_{ikh} = e_{ikh} m_i = 2KV (n_i h_h - n_h h_i) n_l h_l $$

(2.6)

3. In the hydrodynamic description of the suspension as a homogeneous continuous medium, in order to allow for the rotational degrees of freedom of the particles we have to introduce an additional macroscopic variable $\mathbf{S}$, the volume density of the internal moment of momentum. The latter has the sense of the product $\mathbf{I} (\boldsymbol{\omega})$, where $\mathbf{I}$ is the sum of the intrinsic moments of inertia of the spherical particles in unit volume of the suspension, $\langle \boldsymbol{\omega} \rangle$ is the mean angular velocity of their rotation. The stress tensor of a medium with internal rotation contains the antisymmetrical part [6]

$$ \sigma_{ikh} = e_{ikh} \mathbf{\Omega}, \quad \mathbf{\Omega} = (2\tau_S)^{-1} (\mathbf{S} - \mathbf{I} \mathbf{\Omega}) $$

(3.1)

where $\tau_S = a^2 \rho / 15 e_0$ is the relaxation time of the internal moment $\rho$ is the density of the particle. The change in the density of the internal moment of momentum is described by the equation

$$ \frac{dS}{dt} = -\frac{1}{\tau_S} (S - I \mathbf{\Omega}) + M $$

(3.2)