AXIALLY SYMMETRICAL INSTABILITY MODES
IN A CYLINDRICAL SHELL UNDER IMPACT

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An analysis is presented of the interaction between longitudinal and transverse motions of a circular cylindrical shell under impact on the end surface. At infinite and finite velocities of perturbation propagation along the generatrix this analysis reveals the instability modes in the shell which build up fastest and are similar to those revealed if the buckling process at a finite velocity of perturbation propagation were described in the real time of compressive loading action. It is established that a cylindrical shell under intensive loading can be simulated by a rod under longitudinal impact (the similarity parameters are indicated). This conclusion is confirmed by a comparison with experimental results.

Elastic systems are characterized by a selective amplification of certain higher-order instability modes under conditions of high-intensity loading [1]. The effect of a wave process on the buckling of rods and shells with a sudden application of a load to an elastic system has been observed in the experiments in [2-4]. The instability mode has been determined in an asymptotic representation for a semiinfinite rod, assuming a finite velocity of longitudinal perturbations [5] and with the aid of series expansion on a variable interval [6]. A problem analogous to the one which will be considered here has been solved numerically in [7]; probably because of the low impact rate, no wave generation was observed along the shell. An effect of a wave process on the buckling mode is mentioned in [8].

1. Formulation of the Problem. The longitudinal and the transverse motions of a circular cylindrical shell are described by the following system of equations:

\[
\begin{align*}
\frac{D}{R} \left( \frac{w_{xxx}}{R^2} + \frac{12}{R^2 h^2} \right) - \frac{\nu E h}{R (1 - \nu)} u_x + (N w_x)_x + \rho h w_{tt} &= f (x, t) \\
N_x - c^2 N_{tt} &= \varphi \left( \frac{h}{R} \right) w_{tt}
\end{align*}
\] (1.1, 1.2)

Here \( x, \ t \) are the longitudinal coordinate and the time; \( u, w \) are the longitudinal and the transverse displacement of the mean shell surface with the radius \( R \); the subscripts refer to differentiation with respect to the respective variables; \( E, \nu, \rho \) are the Young modulus, the Poisson ratio, and the material density; \( D \) is the cylindrical rigidity; \( c = \left( \frac{E h}{\rho (1 - \nu^2)} \right)^{1/2} \) is the velocity of sound; \( f(x, t) \) is a function defined by perturbations or imperfections; and \( N \) is the longitudinal force, defined in the linear approximation by the equation

\[
N = \frac{E h}{(1 - \nu^2)} \left( u_x - \nu w / R \right)
\] (1.3)

The initial and the boundary conditions for the impact state at the time of loading a semiinfinite hinge-supported shell which had been at rest before the impact are

\[
\begin{align*}
w &= w_1 = 0 \quad (t = 0, 0 < x < \infty), \quad w = w_{xx} = 0 \quad (x = 0) \\
N &= N_0 = \text{const} \quad (t > 0, x = 0), \quad N = N_1 = 0 \quad (t = 0, x > 0)
\end{align*}
\] (1.4)
During the loading of the shell there may appear small transverse loads, the force $N_0$ may be applied eccentrically, and also the shape of the mean shell surface may differ from an ideal cylindrical one. All this is accounted for by function $f(x, t)$, which will be assumed given.

Equations (1.1) and (1.2) are identical to those known in § 215 [9] from the theory of vibrations of circular cylindrical shells, where the expressions for the forces at the mean surface have been linearized through superpositions.

The solution of problem (1.1), (1.2), (1.4) becomes much simpler when the longitudinal force $N(x, t) = \text{const}$. This can be realized in two cases.

1) If the shell wall is sufficiently thin ($h/R \ll 1$) to make the expression on the right-hand side of (1.2) negligibly small: then the solution to the simplified wave equation with respect to the longitudinal force for the shell under conditions (1.4) becomes

$$N = N_0 = \text{const}$$

2) If the perturbations are propagated along the shell at an infinite velocity; then we have from the equation of motion

$$u_{xx} - w_{xt}/R = u_{tt}/c^2$$

that $N = N_0 = \text{const}$ at $c \to \infty$ (see (1.3)).

2. Analysis of Buckling Modes under a Constant Force. We consider the equation of dynamic instability for a shell:

$$D \left[ w_{xxxx} + 12 (1 - v^2) R^{-2} h^{-2} w_x + N_0 w_x + \rho h w_{tt} \right] = f^*(x, t)$$

(2.1)

This equation has been obtained by eliminating $u_x$ from (1.1) with the aid of (1.3), and $f^*(x, t)$ denotes those components which are independent of $w$.

The initial and the boundary conditions for Eq. (2.1) are ($L$ is the shell length)

$$w = w_t = 0 \quad (t = 0, 0 \leq x \leq L), \quad w = w_{xx} = 0 \quad (x = 0, L)$$

(2.2)

The solution to problem (2.1), (2.2) is sought in the form

$$w = \sum_{m=1}^{\infty} q_m(t) W_m(x), \quad W_m(x) = \sin \frac{m\pi x}{L} \quad (m = 1, 2, \ldots)$$

(2.3)

Here $W_m$ are the instability modes.

We now analyze the behavior of a shell under intensive loading, i.e., under $N_0 > N^*$ with $N^* = 4[3(1 - \nu^2)]^{1/2} DR^{-1} h^{-1}$ being the critical Euler load.

Of interest during the action of high-intensity loads $N_0 > N^*$ are those degrees of freedom in the system (instability modes) which correspond to a fast increase of deflections [1]. The fastest growing instability mode increases exponentially with the index $\alpha$, with

$$\alpha^2 = \frac{12 (1 - v^2)}{R h^4} \frac{D}{\rho h} (\eta^4 - 1), \quad \eta^* = \frac{N_0}{N^*}$$

(2.4)

Index $\alpha_0$ of the fastest growing mode in a rod is

$$\alpha_0 = \frac{n^4 E l}{4\xi^2 R h} \eta^4$$

(2.5)

The designations are the same here as in [1]. The instability-mode numbers for (2.4) and (2.5) are respectively

$$m_0^2 = \frac{2n^2 [3(1 - v^2)]^{1/2} L^2}{R h}, \quad m_0^2 = \frac{n^4}{2}$$

(2.6)