PLANE PROBLEMS OF AEROTHERMOPTICS

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An exact solution is obtained of the refraction problem in a two-dimensionally inhomogeneous medium. Light propagation in nonisothermal gas streams is examined, and the influence of heat exchange on the light beam trajectory and intensity distribution is investigated.

In connection with the need to produce optical waveguides for optical communications systems, the production of thermogasdynamic waveguides is of great value [1-3]. Light propagation in inhomogeneously heated laminar subsonic (M ≤ 0.2) gas streams, which it is proposed to use to control light beams, is considered in this paper.

1. Light propagation in a medium is described by a system of Maxwell equations (secondary charges and currents are assumed absent). The inhomogeneity of the dielectric permittivity field (for optical frequencies ν = 1 [4]) is determined by convection which depends on absorption of radiation by the medium. Light beams in which the fields E and H are much less than the intermolecular fields, and moreover, whose radiation energy is much less than the internal gas energy E^2 << 8πC_vT (C_v is the isochoric specific heat per unit volume in the system of units) are considered. For air at room temperature we should have E << 3*10^3 W/cm or the radiation power <<1 GW/m^2. It is then sufficient to limit oneself to a linear relationship between E and H in the fixed gas and electrostriction and the electrocaloric effect can be neglected (which would result in excessive accuracy if taken into account since the gas is considered incompressible). The gas incompressibility also results in the absence of anisotropy ε because of the inhomogeneity of the gas velocity (the Maxwell dynamo optics effect). If the frequency dispersion ε of the gas at rest is negligible in the frequency band under consideration and there is no spatial dispersion, then the spatial dispersion ε in a moving medium which occurs because of field entrainment by the medium can be neglected. Therefore, the radiation does not influence convection, but convection influences the radiation because of the dependence of ε on the thermodynamic gas flow parameters.

To the accuracy of quantities on the order of vc^-1, the material equations in a moving medium have the form [4]

\[ D = εE + \frac{ε - 1}{c} [VH], \]

\[ B = H + \frac{ε - 1}{c} [EV]. \]

In gases of nonpolar molecules (in dry air, for example) [5]

\[ ε = 1 + 4παN. \]

Since 4παN << 1 (for air ε = 1 = 10^-3), then to the accuracy of quantities of the order 4παNvc^-1, we obtain D = εH, B = H from (1) and (2). To the same accuracy, the light phase velocity in a moving medium without dispersion can be considered invariant [4]. It is sufficient to consider the two-dimensional system of Maxwell equations for light rays being propagated along plane inhomogeneities. In first-order perturbation theory in 4παN, the Cartesian E and H components satisfy the scalar wave equation

\[ \frac{∂^2U}{∂x^2} + \frac{∂^2U}{∂y^2} = \frac{ε(x, y)}{c^2} \frac{∂U}{∂t^2}. \]
To determine the ray characteristics, it is sufficient to limit oneself to (4) without taking account of the vector nature of the fields \( E \) and \( H \). The properties of the characteristics of (4) result in a variational problem for the Fermat functional [6]:

\[
S = \int n ds \rightarrow \text{extr}, \quad n = \sqrt{c},
\]

where \( ds = \sqrt{(dx)^2 + (dy)^2} \) is the ray length element. The Euler equation for the functional (5) (the ray equation) has the form

\[
\frac{d^2 y}{dx^2} = \frac{1}{n} \left( \frac{\partial n}{\partial y} \frac{dy}{dx} - \frac{\partial n}{\partial x} \right) \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right] \tag{6}
\]
or

\[
\frac{d \varphi}{dx} = \frac{\partial \ln n}{\partial y} - \tan \varphi \frac{\partial \ln n}{\partial x}, \quad \frac{dy}{dx} = \tan \varphi(x, y). \tag{7}
\]

The angle of refraction depends on \( n \), hence it follows from the first equation in (7) that

\[
\frac{\partial \varphi}{\partial \ln n} = \cot \varphi + \gamma, \quad \frac{\partial n}{\partial x} \left( \frac{\partial n}{\partial y} \right)^{-1} = \tan \gamma(x, y).
\]

from which

\[
\varphi = \arccos \left[ \frac{n_x}{n} \cos (\varphi_e + \gamma_e) \right] - \gamma \tag{8}
\]

(the subscript \( e \) refers to the initial point of ray incidence). Equations (8) and (9) can be rewritten in the form

\[
\frac{\partial \varphi}{\partial \ln n} = -\tan(\varphi - \xi), \quad \cot \xi = \tan \gamma \tag{10}
\]

where \( n = n(x) \)

\[
\varphi = \arcsin \left[ \frac{n_x}{n} \sin (\varphi_e - \xi_e) \right] + \xi.
\]

where \( \varphi = \arcsin \left( n_e n^{-1} \sin \varphi_e \right) \) for \( n = n(x) \) and \( \varphi = \arccos \left( n_e n^{-1} \cos \varphi_e \right) \) for \( n = n(y) \). Relationships (9) and (10) are meaningful only for \( |n_e n^{-1} \cos (\varphi_e + \gamma_e)| \leq 1 \) or \( |n_e n^{-1} \sin (\varphi_e - \xi_e)| \leq 1 \); therefore, \( n_e \leq n \). Hence, a ray is propagated toward increasing \( n \). Let us note the strong dependence of the angle of refraction \( \varphi \) on the ratio of the components of the gradient of \( n \). The dependence of \( \varphi \) on \( n \) is less essential since \( 4\pi nN \ll 1 \). In an ideal gas \( p = NkT \) and because of (3) and (5)

\[
n \approx 1 + \frac{rp}{T}, \quad r = 2\pi nk^{-1} \tag{11}
\]

The relative change in density due to the change in pressure is small for a subsonic stream and for \( M \ll 0.2 \) it does not exceed 2% [7]. Therefore, the inhomogeneity in the field \( T \) is the governing factor of the inhomogeneity in \( n \), meaning, the curving of the ray trajectory. Therefore,

\[
n = 1 + \frac{b}{T}, \tag{12}
\]

where \( b = rp = \text{const} \) (for air under normal conditions \( b = 9 \times 10^{-2} \) K). It is proposed to produce a field inhomogeneity because of the change in \( T \) as well as \( N \) in gas lenses. However, if there are such heat and mass transfer problems, it is possible to limit oneself to the consideration of one of the factors and just the temperature inhomogeneity of \( n \) will henceforth be considered.

The amplitude dependences of the light wave field are conveniently studied by substituting \( U = W(x, y) \exp (i\omega t) \) into (4). Then

\[
\Delta W + k_0^2 n^2 W = 0, \tag{13}
\]