COMBINED HEAT AND MASS TRANSFER DURING ABSORPTION IN DROPS AND FILMS

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Combined heat and mass transfer during absorption in films and drops is discussed. Using an approximate approach, simple analytic relations are obtained for the basic characteristics of the process.

The physical processes which take place during absorption by liquid drops or films moving in a medium containing an absorbable material are extremely diverse and complex. In studying them it is therefore rather useful to have rather simple and predictable models which give a qualitatively true reflection of them although failing to reproduce the detailed pattern of the phenomena.

We consider the problem of combined heat and mass transfer during absorption of vapor by drops of fluid under the following assumptions. The atmosphere in which the drop is located contains no noncondensable gases. The drop is stationary with respect to the surrounding vapor atmosphere and is spherical in shape. At the initial time \( t = 0 \), a state of saturation is established instantaneously over the entire surface of the drop for the absorbed-material–liquid–solution system which is subsequently maintained throughout the entire process.

The dependence of concentration \( C \) on temperature \( T \) in the state of saturation is linear, \( C = dT + b \).

All physical parameters of the problem (coefficients of thermal conductivity, diffusion, etc.) are constant over the ranges of temperatures and concentrations considered.

Under the assumptions made, heat and mass transfer is described by the following system of equations:

\[
\frac{\partial T}{\partial t} = a \left( \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right),
\]

\[
\frac{\partial C}{\partial t} = D \left( \frac{\partial^2 C}{\partial r^2} + \frac{2}{r} \frac{\partial C}{\partial r} \right)
\]

with boundary conditions of the form

\[
C(t, R) = dT(t, R) + b,
\]

\[
\lambda \frac{\partial T}{\partial r} \bigg|_{r=R} = \rho_r D \frac{\partial C}{\partial r} \bigg|_{r=R}
\]

We take

\[
T(0, r) = T_0, \quad C(0, r) = C_0
\]

as initial conditions for Eqs. (1), where \( C_0 \) is less than the saturation value corresponding to the temperature \( T_0 \).

We also assume that the process of heat and mass transfer is localized near the surface of the drop within the limits of a layer with a thickness less than the radius of the drop.


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Fig. 1. Profiles of temperature (dashed lines) and concentration (solid lines) at various cross sections of a film (Le = 25; Tw/To = 0.5; dTo/Co = -0.5; b/Co = 1.7; Ka = -0.1): I) x = 0.03; II) x = 0.09; III) x = 0.21; IV) x = 0.3.

The possibility of a surface-layer approximation then follows. In that case, the initial conditions transform into boundary conditions at the boundary of an asymptotic surface layer, i.e., one can write

\[ T_{r \to \infty} = T_0, \]
\[ C_{r \to \infty} = C_0. \] (4)

For the surface-layer approximation we also assume

\[ C(t, R) = C_s(t); \] (6)
\[ T(t, R) = T_s(t). \] (7)

The temperature and concentration at the surface of the drop (\( T_s, C_s \)) are determined during the solution of the problem.

By means of the substitutions \( \tilde{C} = (C - C_0)r \) and \( \tilde{T} = (T - T_0)r \), the system (1) with the boundary conditions (4)-(7) reduces to a problem allowing a self-similar solution of the form

\[ \tilde{C} = \frac{2R(C_s - C_0)}{V \pi} \int_0^{R_s} \exp(-x^2) dx + R(C_s - C_0), \]
\[ \tilde{T} = \frac{2R(T_s - T_0)}{V \pi} \int_0^{R_s} \exp(-x^2) dx + R(T_s - T_0). \]

Here \( C_s \) and \( T_s \) are considered constant during the integration. In the dimensionless variables

\[ \tilde{C} = \frac{C}{C_0}; \quad \tilde{T} = \frac{T}{T_0}; \quad \text{Fo} = \frac{a}{R^2} t; \quad x = \frac{r}{R}, \]

this solution takes the form

\[ \tilde{C} = \frac{C_s - 1}{x} \text{erf} \left[ \frac{V \text{Le} (x - 1)}{2V \text{Fo}} \right] + \frac{C_s - 1}{x} + 1; \] (8)

\[ \tilde{T} = \frac{T_s - 1}{x} \text{erf} \left[ \frac{x - 1}{2V \text{Fo}} \right] + \frac{T_s - 1}{x} + 1. \] (9)

We obtain the following expressions for \( \tilde{T}_s \) and \( \tilde{C}_s \) from conditions (2) and (3):

\[ \tilde{T}_s = \frac{C_0 - b}{dTo} \left( \frac{V \text{Le}}{V \pi \text{Fo}} - 1 \right) - \frac{C_0 \text{Le}}{r_0 d} \left( \frac{1}{V \pi \text{Fo}} - 1 \right); \] (10)
\[ \tilde{C}_s = \frac{dTo}{C_0} \tilde{T}_s + \frac{b}{C_0}. \] (11)