VARIATION OF PRESTRESS IN FILAMENT-WOUND GLASS-REINFORCED PLASTICS

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The authors examine the process of winding prestressed parts from oriented glass-reinforced plastics. It is shown that the essential anisotropy of the properties of these materials is responsible for variation of the specified prestress. The law of distribution of the tensile forces is investigated in the linear-elastic approximation for the case of a ring wound onto a rigid mandrel. A method of calculation is proposed that permits the change in prestress to be estimated and gives the critical number of turns beyond which the pressure on the mandrel ceases to increase.

1. In filament-winding each successive turn deforms the underlying layers in the radial direction. This leads to a change in the initial tensioning law $T_0(r)$.

![Fig. 1. Pressure on mandrel $k\Sigma T$ wound with: 1) isotropic material (nickel tape), 2) essentially anisotropic material (LSB-F glass-reinforced plastic). $n$ is the number of turns.](image)

When isotropic materials, e.g., steel tape, are wound onto a rigid mandrel, the variation of tension in the individual layers is negligibly small. The winding of essentially anisotropic materials, whose modulus of elasticity in the direction of winding considerably exceeds the modulus in the perpendicular direction, is accompanied by a considerable variation of the initial tensioning law over the cross section of the wound part, which it is impossible to ignore.

As will be shown below, during winding (before polymerization of the resin) oriented glass-reinforced plastics are essentially anisotropic materials. Experiments have shown that the winding of such materials is accompanied by a fall in the winding force over the cross section; the actual total prestress, retained in the finished article (Fig. 1),* is considerably less than that originally specified [1, 2], and the strength decreases with increase in the number of turns [3, 4]. Consequently, it is necessary to take into account the essential anisotropy of glass-reinforced plastics in solving winding problems. However, this is often overlooked in determining the optimal winding conditions [5].

We have investigated the law of distribution of tensile forces over the cross section during winding of ring-shaped prestressed parts made of oriented glass-reinforced plastic. The problem is assumed to be axisymmetric; the successive turns of glass-reinforced plastic are replaced by thin rings. Thus, the problem reduces to the investigation of the state of stress of a system composed of a large number of thin anisotropic rings fitted over a mandrel and then one over the other with a tightness equal to the prestress of an individual layer. The problem is solved in the linear-elastic formulation; the winding force $T_0 = \text{const}$ and the mandrel is assumed to be absolutely rigid.

It should be noted that in the direction of winding, glass-reinforced plastics obey Hooke's law with sufficient accuracy. In a direction perpendicular to the direction of winding, the deformation properties are determined by the resin, and for a series of layers of glass-reinforced plastic tape in compression linearization of the $\sigma-\varepsilon$ relation is only a first approximation. The proposed method makes it possible to solve the problem even for a nonlinear $\sigma-\varepsilon$ relation in compression.

![Fig. 2. Model of multilayer winding.](image)

2. We shall treat the wound ring as a regular multilayer medium (Fig. 2), composed of alternating "hard" layers of glass fiber and "soft" intermediate layers of resin. The same figure shows the thickness of the layers and the dimensions of the ring and mandrel. The large number of layers and their small absolute thickness make it possible to replace the multilayer medium with an essentially anisotropic continuous medium [6–8]. To solve the problem of the distribution of the winding force over the cross section of the ring it is necessary to consider the problem of determining the radial displacements in an anisotropic ring fitted

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*The experimental data shown in Fig. 1 were obtained by R. E. Brivmanis on the apparatus described in [1].
without play or tightness over a rigid mandrel and loaded over the outer surface by a uniformly distributed pressure \( T_0/r_e \) (see Fig. 2). The solution of this problem was obtained in [9]; the expression for the radial displacements takes the form:

\[
\Delta u_r = -\frac{MNT_0 \left( \frac{r_e}{R_1} \right)^\beta \left( r_e^2 - R_1^2 \right)}{
\left( \frac{r_e}{R_1} \right)^{2\beta} - N} \left( \frac{r^2 - R_1^2}{r_e^2 R_1^2} \right) \ldots, \quad (1)
\]

where \( M = \frac{a_0 \beta - a_r}{\beta} \); \( \beta = 1 \left( \frac{a_r}{a_0} \right) \); \( N = \frac{a_0 \beta + a_r}{\beta} \); \( a_r = \frac{1}{E_r} \); \( a_0 = -\frac{1}{E_0} \); \( \frac{a_0}{E_0} = -\frac{a_0}{E_r} \); \( T_0 \) is the winding force.

The deformation properties of the homogeneous medium, obtained by passing to the limit [7,8], are determined by the elastic characteristics of the glass fiber \((E_g)\) and the resin \((E_r)\) and their percentage content:

\[
\begin{align*}
\alpha_0 &= \frac{1}{E_0} = \frac{1}{E_g h} \quad \text{and} \quad \alpha_r = \frac{1}{E_r} = \frac{1}{E_r (h+1)}.
\end{align*}
\]

Experiments showed that \( E_g \gg E_r \). For example, for wound AG-4S glass-reinforced plastic \( E_g/E_r \approx 3000 \). This means that glass-reinforced plastics in the wound state can be treated as essentially anisotropic materials. The level anisotropy makes it possible to simplify the problem; we can assume that in compression in the radial direction the transverse strains of the glass fiber can be neglected as compared with the intermediate resin layer. The circumferential force is resisted only by the glass fiber. In this case the change in tensile stress in each elementary layer is due only to "sagging" of the underlying layers. The introduction of these assumptions is equivalent to neglecting the terms \( a_0 \beta \) as compared with \( a_1 \) in the expressions for \( M \) and \( N \).

With these assumptions the expression for the radial displacements of an essentially anisotropic shell takes the form:

\[
\Delta u_r = -\frac{T_0}{E, \beta} \left( \frac{r_e}{R_1} \right)^\beta \left( r_e^2 - R_1^2 \right) \left( \frac{r^2}{R_1^2} + 1 \right) \frac{r^2 R_1^2}{\beta} \ldots, \quad (2)
\]

where \( \beta = \sqrt{E_g/E_r} \) is a parameter characterizing the anisotropy of the properties of the wound material.

3. We now turn to the determination of the law of variation of the tensile force over a vertical section of the ring. We note that a problem similar to that examined here arises in determining the pressure on a drum around which a cable is wound [10]. The proposed method of solution is simpler. The winding process can be regarded as the successive fitting one over the other of thin rings of thickness \( c = h + t \) with a tightness equal to the tension in the tape. The action of each successive ring on the underlying layers, which are assumed to constitute a homogeneous anisotropic body, is manifested in the development of an external pressure with intensity \( T_0/r_e \) \( (r_e \) is the variable radius of the shell on which the external load acts).

In order to determine the radial displacement \( u_r \) in the wound ring at radius \( r \), it is necessary to take into account its sag due to fitting of all the subsequent rings from \( r_e = r \) to \( r_e = R_e \); \( r \) is the fixed radius at which the displacements are determined, \( r_e \) is the variable outside radius of the shell (see Fig. 2). With this object we sum the displacements obtained at radius \( r_e \) due to the external pressure \( q_e = T_0/r_e \) in rings with outside radii \( r_e = r \); \( r_e = r + c \); \( r_e = r + 2c \); \ldots; \( r_e = R_e \).

Hence

\[
\begin{align*}
u_r &= \Delta u(r_e = r) + \Delta u(r_e = r + c) + \Delta u(r_e = r + 2c) + \ldots + \Delta u(r_e = R_e).
\end{align*}
\]

In view of the thinness of the elementary layer the sum (3) can be approximated by the integral

\[
\begin{align*}
u_r &\approx \Delta u(r_e = r) + \frac{1}{c} \int r_e = r + c \Delta u(r_e) d(r_e).
\end{align*}
\]

The expression for the latter integral is extremely clumsy. Using the fact that for actual structures the ratio \( R_e/R_1 \) lies within the limits \( R_e/R_1 = 1.02-1.10 \), we can write

\[
\begin{align*}
\int_{r_e = r}^{r_e = r + c} \left( \frac{r_e}{R_1} \right)^\beta \frac{d \left( \frac{r_e}{R_1} \right)^\beta}{1 + \left( \frac{r_e}{R_1} \right)^\beta} &\approx \frac{1}{\beta} \int_{r_e = r}^{r_e = r + c} \left[ \left( \frac{r_e}{R_1} \right)^\beta \right]^{\frac{1}{\beta}} d \left( \frac{r_e}{R_1} \right)^\beta = \\
&= \frac{1}{\beta} \arctg \left( \frac{r_e}{R_1} \right)^\beta \Bigg|_{r_e = r}^{r_e = r + c} \ldots.
\end{align*}
\]

In order to estimate the error thus introduced, the exact and approximate values of the integral (5) were computed on a BESM-2 computer for \( 1.01 \leq R_e/R_1 \leq 1.1 \) and \( 10 \leq \beta \leq 80 \). It was found that the difference between the exact and approximate values does not exceed 10% within the above-indicated limits of variation of the parameters \( \beta \) has practically no effect on the accuracy of the approximation. At \( R_e/R_1 = 1.05 \)