The basic characteristics of heat transfer for flow in a channel with a nonplanar magnetic-liquid coating are studied by means of numerical calculations.

The analytical investigations performed in Part 1 of this work demonstrated that it is in principle possible to reduce the hydraulic resistance and at the same time intensify heat transfer in a laminar flow in a channel by coating the channel walls with a low-viscosity magnetic-liquid [1]. In the case of one-dimensional flow heat transfer is intensified mainly by means of the smoothing of the velocity profile of the main flow owing to the reduction of the tangential stresses at the interface of the liquids. As a result for a given flow rate the velocity of the liquids near their interface increases, and this results in intensified refreshment of the media and intensification of heat transfer. A low-viscosity magnetic-liquid coating changes the structure of the flow in the channel in such a manner that heat transfer is intensified and at the same time the pressure losses are lower than in the case of uncoated channel. In [1] it was assumed that the form of the interface between the main flow and the coating can be assumed to be flat. This assumption is valid if sufficiently stringent restrictions are imposed on the geometry of the system (the length of the layer of magnetic liquid must be much greater than the width of the channel and the length of the starting thermal section).

In real systems with a magnetic-liquid coating the period of the coating \( A \) is comparable to the width of the channel. Hence in studying the hydrodynamics and heat-transfer processes in such systems the curvature of the interface between the media and the possibility of convective transfer across the channel must be taken into account. The purpose of this work is to investigate such systems and to analyze the mechanisms which are responsible for the intensification of heat transfer and are connected with the presence of a nonplanar interface between the magnetic and nonmagnetic liquids.

A two-dimensional conjugate problem of this type cannot be solved analytically. Because of this the hydrodynamics and heat transfer were investigated with the help of numerical calculations.

The geometry of the problem is shown in Fig. 1. The form of the interface between the media was assumed to be given and was approximated by sinusoid \( \xi(x) = h(\delta + a \cos 2\pi x/A) \). As follows from existing experimental results [2, 3], for a given volume of magnetic liquid and configuration of the magnetic field the form of the surface of the magnetic liquid does not depend significantly on the velocity of the main flow; this is the basis for regarding the form of the interface of the media as an external parameter. This greatly simplifies the problem, since there are significant computational difficulties in solving conjugate problems with an unknown boundary and a more accurate formulation will not significantly affect the final result (the characteristics of heat transfer).

The assumption that the properties of the liquids do not depend on the temperature made it possible to separate the hydrodynamic and thermal parts of the problem. The velocity profile in the heat-transfer zone was assumed to be established; the computed velocity field was used to solve the equations of convective heat transfer.

The hydrodynamic part of the problem was solved using the stream function and the vorticity as the variables. The corresponding equations and boundary conditions have the form

\[ \text{Belorussian Polytechnical Institute, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 59, No. 3, pp. 403-408, September, 1990. Original article submitted October 4, 1989.} \]
Fig. 1. The geometry of the problem.

Fig. 2. The temperature field in the input section of the channel: \( a = 0.15; \, \delta = 0.8; \, \Lambda = 0.5; \, \text{Re}_1 = 100; \, \eta_1/\eta_2 = \lambda_2/\lambda_1 = 5; \, \text{Pr}_1 = 100; \, \text{Pr}_2 = 10.\)

\[
\Delta \psi = -\omega, \quad v_x = \partial \psi / \partial y, \quad v_y = -\partial \psi / \partial x; \\
\text{Re}(v_x \partial \omega / \partial x + v_y \partial \omega / \partial y) = \Delta \omega; \\
y = 1: \, \psi = 1/2, \, \partial \psi / \partial y = 0; \\
y = \xi(x)/h: \, \psi = 1/2, \{\partial \psi / \partial y\} = 0; \left\{ \eta \left( \omega - \frac{2 \text{Pr} \xi''}{1 + \xi'^2} \frac{\partial \psi}{\partial y} \right) \right\} = 0; \\
y = -\xi(x)/h: \, \psi = -1/2, \{\partial \psi / \partial y\} = 0; \left\{ \eta \left( \omega + \frac{2 \text{Pr} \xi''}{1 + \xi'^2} \frac{\partial \psi}{\partial y} \right) \right\} = 0; \\
y = -1: \, \psi = -1/2, \, \partial \psi / \partial y = 0; \\
\psi(x + \Lambda, \, y) = \psi(x, \, y), \, \omega(x + \Lambda, \, y) = \phi(x, \, y).
\]

It can be shown that the functions \( \psi(x, \, y) \) and \( \omega(x, \, y) \) are odd functions of the transverse coordinate \( y \). This property was used in the solution (it was assumed that the domain of \( y \) is \([-1, 0]\)). The condition of periodicity \( \psi \) and \( \omega \) is connected with the steady-state character of the flow of the liquids.

The problem (1) was solved numerically by the method of finite differences using a power-law interpolation function between the nodes [4]. The algebraic equations obtained by casting the system into a finite-difference form was solved by the Gauss-Seidel method with a relative error of \( 10^{-6} \). The pressure losses \( \Delta \rho \) in the channel were determined from the computed fields \( \psi \) and \( \omega \).

An analogous problem was solved numerically in [2]. In so doing a transformation was made to a curvilinear coordinate system, so that the interface of the liquids would be one of the coordinate lines. The results obtained in this work are in good agreement with the results of [2] (the relative deviation is less than 1\%).

The following model was used to study heat transfer accompanying flow in a channel with a sinusoidal magnetic-liquid coating: it was assumed that the temperatures of the walls are equal and constant along the channel and the temperature in the input section of the channel was assumed to be constant and different from the temperature of the walls. The corresponding equations and boundary conditions for the thermal problem have the following form:

\[
\text{Pe}(v_x \partial \Theta / \partial x + v_y \partial \Theta / \partial y) = \Delta \Theta; \quad y = 0: \, \partial \Theta / \partial y = 0; \\
y = -1: \, \Theta = 0; \quad y = -\xi(x)/h: \{\lambda \partial \Theta / \partial n\} = 0, \{\Theta\} = 0; \\
x = 0: \, \Theta = 1; \, x \to \infty: \Theta \to 0.
\]