MODELING THE HEAT TRANSFER WHEN A CHEMICALLY REACTING FLOW PASSES A RIB BY MEANS OF THE FINITE-ELEMENT METHOD

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The finite-element method is used to solve the conjugate problem of heat transfer between a ribbed wall and a chemically reacting flow. The distribution of the velocities, temperatures, and concentrations, as well as the local heat-transfer coefficients and efficiency of the rib, is obtained.

Ribbing heat-transfer surfaces is one of the most widespread and effective methods of intensifying heat transfer. As shown by investigation [1], the distribution of the thermohydraulic parameters in the flow around longitudinally ribbed tube bundles is complex. The temperature difference in the cross section of such bundles may reach several thousand degrees in the initial section.

Increasing the efficiency of ribbing in order to reduce the mass and size characteristics of the ribbed surfaces is an urgent and practically important problem. At present, there are various methods and technologies for the production of ribbed surfaces. In one, a U-shaped groove is welded to the outer surface of a tube in order to produce longitudinal ribbing. The deficiency here is the presence of a technological gap between the base of the groove and the tube surface and the consequent formation of additional thermal resistance.

In engineering practice, the calculation of heat transfer in the rib (and the design of the ribbing) is usually reduced to solving a conductive problem, whereas the complex phenomena of convective heat transfer and hydrodynamics are rarely studied. The influence of convection in this approach to the theory of heat transfer of the rib is taken into account by means of the concept of the heat-transfer coefficient, which is usually taken to be constant over the rib height. It is known that this approach often disagrees with the reality. In addition, the heat-transfer coefficients used in the rib calculation are often determined from approximate estimates.

The investigation of rib heat transfer in the presence of a gap is thus one of the aims of the present work. Since it is impossible to measure experimentally both the temperature drop in the gap and its distribution over the rib height, numerical modeling on a computer is used, on the basis of solution of a system of conservation equations by means of the finite-element method (FEM).

Mathematical Model. Initial Equations

A flow of chemically reacting gas in which the following reactions occur is considered: 1) N₂O₄ = 2NO₂; 2) 2NO₂ = 2NO + O₂; 3) NO₃ = NO₂ + O. The first and third reactions occur in equilibrium, and the second as a reaction occurring at a finite rate [2]. A diagram of the flow and the channel studied is given in Fig. 1.

The transport equations and boundary conditions are written in the following approximation: parabolic flow of Newtonian liquid is considered; leakage of heat along the channel axis is neglected, together with thermodiffusion and barodiffusion; and only the longitudinal component of the velocity vector is taken into account. In this case, the initial system of equations takes the form

\[\rho U \frac{\partial U}{\partial z} = \nabla_x y (\mu \nabla_x U) - \frac{\partial \phi}{\partial z};\]  
(1)

\[\rho c_p U \frac{\partial T}{\partial z} = \nabla_x y (\lambda \nabla_x T) + S_r;\]  
(2)

\[
\rho U \frac{\partial C}{\partial z} = \nabla_{xy} (D \nabla_{xy} C) + R_C.
\]

The continuity equation is expediently used in integral form
\[
\int \rho U \text{d}s = \text{const.}
\]

Since the problem is solved in a conjugate formulation, the system is complemented by the heat-conduction equation
\[
\nabla_{xy}^2 T = 0.
\]

The region of integration of Eqs. (1)-(4) is shown in Fig. 1 and consists of two subregions: the ribbed wall of the channel and the region of moving gas. The flow is symmetric relative to the lines AD, BC, CD.

Because the expressions and procedures for determining the coefficients \( \mu, \lambda, C_p, \) and \( D_C \) of the chemically reacting heat carrier \( \text{N}_2\text{O}_4 \rightarrow \text{NO} \) are complex and unwieldy, and on the assumption that this is not fundamental to the solution of the initial system, they are not presented here, nor are the algorithms for calculating the source terms \( S_T \) and \( R_C \) in Eqs. (2) and (3). They are calculated according to the algorithms and procedures in [2, 3].

Solving the parabolic equations corresponding to the given situation requires boundary conditions at all the boundaries of the region and the values of \( U, T, C \) at the channel inlet. Uniform profiles of all the functions \( U, T, C \) are specified at the channel inlet and Neumann conditions at the symmetry lines; at the outer boundary of the supporting wall AB, provision is made for use of thermal boundary conditions of the first, second, and third kinds. Finally, at the wall-gas heat-transfer surface, conservation conditions for the temperature and normal heat flux are satisfied.

The use of FEM and the Galerkin method leads to a system of nodal differential equations, which may be written in matrix form as follows
\[
[C] \left\{ \frac{d\Phi}{dz} \right\} + [K] \{\Phi\} + \{F\} = 0;
\]
\[
[C] = \sum_{i=1}^{N} [C]^{(i)} = \sum_{i=1}^{N} \int_A [N]^T [N] \text{d}s;
\]
\[
[K] = \sum_{i=1}^{N} [K]^{(i)} = \sum_{i=1}^{N} \int_B [D]^T [D] \text{d}s; \quad i = 1, 2;
\]
\[
\{F\} = \sum_{i=1}^{N} \{F\}^{(i)} = \sum_{i=1}^{N} \left[ \int_C [N]^T ds + \int_{L_i} q[N]^T dL. \right.
\]

For each element of the region, integration and summation over all the elements is undertaken in the usual manner. The values of the quantities appearing in Eqs. (7)-(9) are given in Table 1.

Fig. 1. Diagram of flow and region of integration.