HYDRAULIC DRAG IN TURBULENT COOLANT FLOW IN A POROUS CABLE

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The hydrodynamics and heat transfer in the cooling of an electrical cable by radial coolant filtration are studied.

The constructional scheme of the cable is shown in Fig. 1. There are two coolant channels, of circular and annular cross section. Attachments that completely or partially cover the channel force the coolant to filter in the radial direction through the permeable structure formed by the current-carrying strands, the porous insulation, and the supporting base.

The model of a porous body with equivalent permeability, heat conduction, and heat liberation is used for mathematical modeling of the hydrodynamic and thermal processes in the permeable structure. The current-conducting part is cooled on account of heat transfer at the surfaces and intrapore heat transfer.

The hydrodynamics in the heat exchanger is modeled using channel-averaged equations of mass and momentum balance.

The momentum-balance equation is written for the tube

$$\frac{d}{dx^*} \left( \rho u_T^* v_T^* S_T^* + P_T^* S_T^* \right) = -v_T^* 2n_a^*$$  \hspace{1cm} (1)

and for the annular channel

$$\frac{d}{dx^*} \left( \rho c_r^* v_C^* S_C^* + P_C^* S_C^* \right) = -v_C^* 2n_C^* - v_C^* 2n_C^*.$$  \hspace{1cm} (2)

The rate of transfer through the porous cylindrical wall is determined according to Darcy's law

$$a^* \rho^* v^* = b^* \rho^* v^* = \frac{K^*}{v^* \ln b/a} (P_\tau^* - P_C^*).$$

The mass-balance equations in the channels take the form

$$\frac{d}{dx^*} \left( \rho_\tau^* u_\tau^* S_\tau^* \right) = - \rho^* v^* 2n a^*,$$

$$\frac{d}{dx^*} \left( \rho_C^* u_c^* S_C^* \right) = \rho^* v^* 2n b^*. $$

Next, Eqs. (1)-(5) are brought to dimensionless form, referring the linear dimensions to the length scale $L^* = (S_T^* = S_C^*)/(2\pi)^{1/2}$, the channel cross section to the total cross section $S_C^* = S_T^* + S_C^*$, the density to $\rho^*$, the velocity to $u_\tau^*$, the tangential stress and pressure to $\rho_\tau^* \omega_\tau^2$, and the dimensionless transverse mass flow rate $j = \rho^* v^* a^*$. Let $\omega_i = \rho_i u_i S_i, i = T, C$ denote the proportion of the mass flow rate and $j = \rho^* v^* a^*$ the dimensionless transverse mass flow rate.

The pressure is now eliminated from Eqs. (1)-(5). Differentiating Eq. (3) with respect to $x^*$, assuming constant properties, and neglecting the derivatives of the momentum-flow coefficients, an equation for $j$ is obtained

$$- \frac{1}{KN} \frac{dj}{dx} = 2 \left( \frac{\beta_\tau u_\tau^*}{S_\tau^*} + \frac{\beta_C u_C^*}{S_C^*} \right) j = \frac{\alpha_\tau a^*}{S_\tau^*} + \frac{b u_\tau^*}{S_C^*} + \frac{c u_C^*}{S_C^*}. $$

To calculate the coefficients characterizing the flow structure, the presence of local self-similarity of the flow in regions adjacent to the walls $a, b, c$ is assumed, in the form $u^* = f(y^*, v^* c^*)$ in hydrodynamic similarity variables.

The dynamic shear velocities $u_\tau^* a$ and $u_\tau^* b$ are determined from the integral mass-balance equations in the channels

$$\omega_\tau = \int_0^{a^*} u^{* + r^*} dr^*/(u_C^* N^2),$$

$$\omega_c = \omega_b + \omega_c = \int_{b^*}^{c^*} u^{* + r^*} dr^*/(u_C^* N^2) + \int_{c^*}^{a^*} u^{* + r^*} dr^*/(u_C^* N^2).$$

The relation for $u_\tau^* c$ is obtained from the momentum-balance equation in the region $c (f \leq r \leq c)$:

$$\int_f^c \frac{\partial}{\partial x} (ur) dr - u_f \int_f^c \frac{\partial}{\partial x} (ur) dr = - \frac{dP_C}{dx} - a \omega_c.$$  

The left-hand side of Eq. (9) is expressed in terms of the hydrodynamic similarity variables and transformed by integration by parts

$$\int_f^c 2u \frac{\partial}{\partial x} (ur) dr - u_f \int_f^c \frac{\partial}{\partial x} (ur) dr =$$

$5 / 2 2 4,$

$C^* x I_L$