The interior inverse problem of heat conduction for determining the variable coefficient of thermal conductivity and the specific volume heat capacity is studied.

The state vector of a thermal system cannot be determined without reliable data on the thermophysical characteristics (TPC) of the constituent material of the object studied. Such data can be obtained by solving the interior inverse problems of heat conduction (IPH).

The construction of a correct computational algorithm for solving the interior IPH in principle requires information on the nature of the changes of the TPC. Such information, however, is either lacking or is of a very approximate character. Because of this, the polynomial representation of the TPC followed by a search for the constant coefficients of the polynomials approximating the dependences sought, is most widely used. This approach for identifying the coefficients of thermal conductivity of materials using the method of optimal filtering was used in [1], where the constant coefficients of the indicated polynomials were found with quite high accuracy.

The desirability of approximating the characteristics sought by polynomials or some other method (for example, splines) depends to a very large extent on the error of such an approximation, which in its turn is determined by the number of observation points. In addition, increasing the accuracy of the approximation (raising the order of the polynomial) can cause a loss of stability of the estimates obtained in the solution of the IPH. For this reason, it is often better to use the identification which we used previously for determining the boundary conditions of heat transfer [2] and which does not require a preliminary approximation of the dependences sought (separate values of the functions sought are determined at each moment in time, and the dependence being identified is constructed on the basis of these values). We shall call it a pointwise identification.

We shall study below the pointwise identification of the TPC, when the values of the parameters sought are found at each of the nodes of a spatial grid. The method of optimal dynamic filtering, more precisely, its iterative and noniterative modifications, is used.

In this approach the uniqueness of the solution can be guaranteed, if the number of measurements is equal to the number of nodes in the grid. This, naturally, requires a larger computer memory and a faster computer, which cannot always be obtained.

To lower the indicated requirements it is proposed that each time step only one or several values of the function sought, corresponding in some definite manner to the temperatures chosen at each step, be obtained. The method for selecting these temperatures is established by prior study, in the process of which the boundary conditions, the form of the region, and their effect on the temperature field are analyzed. Depending on the nature of the process and the forecasted temperature distribution, in the presence of a sharp nonuniformity of the temperature field some average integral temperatures (from a number of subregions with relatively uniform distribution of temperatures in them) and in the presence of a uniform temperature field the average integral temperature over the entire volume of the body can be used as the determining parameters. The first case requires several (based on the number of subregions) observation points; the second case requires only one observation point. As a result of the solution the dependences $T_{av}(T)$, $\lambda(T)$ and (or) $C_v(T)$, which enable constructing the characteristics sought $\lambda(T)$ or $C_v(T)$.
The two described approaches to the solution of the interior IPH were compared for test problems. In the first case the characteristic sought was represented as \( \lambda(T) = \sum_{m=0}^{N} L_m T^m \).

Then the equation of heat conduction and the boundary conditions (BC) of the second and third kind were written in the finite difference form as follows (to simplify the notation, the one-dimensional case with a uniform grid is considered):

\[
\left[ \frac{-C_v(T_i)h}{\Delta t} \right] (T_i)_{h+1} + \frac{1}{h} \sum_{m=0}^{N} \left[ (T_{i+1})_h - (T_i)_{h+1} \right] \times \\
\left[ \frac{hC_v(T_b)h}{2\Delta t} \right] (T_b)_{h+1} + \frac{1}{h} \sum_{m=0}^{N} \left[ (T_b)_{h+1} - (T_b)_{h} \right] = q_{h+1} + \left[ \frac{hC_v(T_b)h}{2\Delta t} \right] (T_b)_{h};
\]

\[
\left[ \alpha_{h+1} + \frac{hC_v(T_b)h}{2\Delta t} \right] (T_b)_{h+1} + \frac{1}{h} \sum_{m=0}^{N} \left[ (T_b)_{h+1} - (T_b)_{h} \right] \times \\
\left[ \frac{hC_v(T_b)h}{2\Delta t} \right] (T_b)_{h+1} = \alpha_{h+1} (T_m)_{h+1} + \left[ \frac{hC_v(T_b)h}{2\Delta t} \right] (T_b)_{h}.
\]

The equations used in the identification of \( C_v(T) = \sum_{m=0}^{N} M_m T^m \) can be written down in an analogous manner. The expressions for forecasting the coefficients sought in the problems studied have the form \( (L_m)_{h+1/h} = (L_m)_{h/h} \) and \( (M_m)_{h+1/h} = (M_m)_{h/h} \) with \( m = 0, 1, 2 \ldots, N \), and instead of the unknown temperatures \( (T_i)_{k+1/h} \), and instead of the unknown temperatures \( (T_i)_{k+1/h} \) entering into the coefficients in front of \( L_m \) and \( M_m \) the forecast of these temperatures from the preceding time step \( (T_i)_{k+1/h} \) (or the result of the preceding iteration in the iterative filter) is used.

The algorithm of pointwise identification presupposes that the starting finite difference model of the thermal system is first transformed. In identifying \( \lambda(T) \) Eqs. (1)-(3) are converted into the form

\[
\left[ \frac{-C_v(T_i)h}{\Delta t} \right] (T_i)_{h+1/k+1} + \frac{1}{h} \sum_{m=0}^{N} \left[ (T_{i+1})_h - (T_i)_{h+1} \right] \times \\
\left[ \frac{hC_v(T_b)h}{2\Delta t} \right] (T_b)_{h+1/k+1} = q_{h+1} + \left[ \frac{hC_v(T_b)h}{2\Delta t} \right] (T_b)_{h/k};
\]

\[
\left[ \frac{hC_v(T_b)h}{2\Delta t} \right] (T_b)_{h+1/k+1} = \alpha_{h+1} (T_m)_{h+1/k+1} + \left[ \frac{hC_v(T_b)h}{2\Delta t} \right] (T_b)_{h/k};
\]

\[
\left[ \alpha_{h+1} + \frac{hC_v(T_b)h}{2\Delta t} \right] (T_b)_{h+1/k+1} \times \\
\left[ \frac{hC_v(T_b)h}{2\Delta t} \right] (T_b)_{h+1/k+1} = \alpha_{h+1} (T_m)_{h+1/k+1} + \left[ \frac{hC_v(T_b)h}{2\Delta t} \right] (T_b)_{h/k},
\]

and in the identification of \( C_v(T) \) they are converted to the form