ORTHOTROPIC PLATE WITH INCLUSION HEATED BY A HEAT SOURCE


UDC 536.24

The article presents solutions of steady problems of heat conduction for an orthotropic plate with foreign inclusion of arbitrary and small thickness.

We consider an orthotropic plate with thickness 2δ with an inclusion in the form of a strip of width 2h. We represent the thermophysical characteristics of the system under examination in the form

\[ p(x) = p^{(0)} + (p^{(1)} - p^{(0)}) N(x), \]

where \( p^{(0)} \) and \( p^{(1)} \) are the characteristics of the inclusion and of the base material, respectively, \( N(x) = S_a (x+h) - S_a (x-h), S_a (x) \) are asymmetric unique functions [1]. Heat exchange with the environment is effected through the surfaces \( z = \pm \delta \) according to Newton's law. For determining the temperature we have the equation [2]

\[ \frac{\partial}{\partial x} \left[ k_x (x) \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[ k_y (x) \frac{\partial T}{\partial y} \right] - \frac{\alpha (x)}{\delta} T = -w. \]

Heating of a Plate by a Linear Heat Source. We assume that an infinite orthotropic plate with an inclusion in the form of a strip 2h wide is heated by a linear heat source of intensity \( q \), situated at the center of the inclusion. To determine the stationary temperature field, we have Eq. (2), where \( w = \frac{q}{2\delta} \delta(x) \delta(y) \), and the boundary conditions

\[ \lim_{|x| \to \infty} T = 0. \]

Taking (1) into account and using the formula

\[ (\psi \psi)' = \psi' \psi + \psi \psi' = [\psi \psi] \delta_\pm (x-x_i), \]
we obtain

\[
\frac{\partial^2 T}{\partial x^2} + [k_y^{(1)} + (k_y^{(0)} - k_y^{(1)}) N(x)] \frac{\partial^2 T}{\partial y^2} + (1 - K_x^{-1}) \times \\
\times \left[ \frac{\partial T}{\partial x} \right]_{x=-h=0} \delta_+(x + h) - \frac{\partial T}{\partial x} \left[ x = h \right] - \delta_-(x - h) - \lbrack x_1^2 + (x_0^2 + x_1^2) N(x) \rbrack T = -Q \delta(x) \delta(y),
\]

where \([\varphi], [\psi] \) are the jumps of the functions \( \varphi \) and \( \psi \), respectively, at the point \( x_1 \), \( k_y^{(n)} = \lambda_y^{(n)}/\lambda_x^{(n)}, x_1^2 = \sigma_x^{(n)}/\delta \lambda_x^{(n)} \) \( (n = 0, 1) \), \( K_x = \lambda_x^{(0)}/\lambda_x^{(1)} \); \( Q = q/2 \delta \lambda_x^{(0)} \); \( \delta_+(\xi) = S_{+}(\xi) \).

We multiply Eq. (5) by \( N(x) \) and introduce the substitution \([3, 4]\)

\[\Theta = TN(x).\]

Then for determining the function \( \Theta \) we obtain the equation

\[
\frac{\partial^2 \Theta}{\partial x^2} + k_y^{(0)} \frac{\partial \Theta}{\partial y^2} - \lambda_x^{(0)} \Theta = K_x^{-1} \left[ \frac{\partial T}{\partial x} \right]_{x=-h=0} \delta_+(x + h) - \\
\times \left[ \frac{\partial T}{\partial x} \right]_{x=h=0} \delta_-(x - h) + T \left[ x = h \right] \delta_+(x + h) - \\
\times \delta_+(x - h) - Q \delta(x) \delta(y).
\]

Applying the integral Fourier transformation with respect to \( y \) to (7), we write

\[
\frac{d^2 \Theta}{dx^2} - \gamma_0^2 \Theta = K_x^{-1} \left[ T_+ = T_{-}(x - 2h) \delta_+(x + h), \right. \\
\left. T_- = T_{-}(x + 2h) \delta_-(x - h), \gamma_0^2 = k_y^{(0)} \eta^2 + x_0^2 \right].
\]

The solution of this equation is as follows:

\[\overline{\Theta} = K_x^{-1} (P_1^+ - P_1^-) + (1 - K_x^{-1}) (P_2^- - P_2^+) - \frac{Q}{V 2\pi} \frac{\text{sh} \gamma_0 x}{\gamma_0} S(x),\]

where

\[
P_1^\pm = \frac{1}{\gamma_0} \left\{ \frac{d}{d\xi} [\overline{T}(\pm 2h - \xi) \text{sh} \gamma_0 (\xi - x)] \right\}_{\xi = \pm h = 0} S_{+}(x \mp h);
\]

\[
P_2^\pm = \frac{1}{\gamma_0} \left\{ \text{sh} \gamma_0 (x \mp h) S_{+}(x \mp h); S(x) = \begin{cases} 1, x > 0, \\ 0.5, x = 0, \\ 0, x < 0. \end{cases} \right. \]

Taking (9) into account, we have the following equation for determining \( \overline{T} \):

\[
\frac{d^2 \overline{T}}{dx^2} - \beta^2 \overline{T} = K_x^{-1} (\gamma_0^2 - \beta^2) (P_1^+ - P_1^-) + (1 - K_x^{-1}) (\gamma_0^2 - \beta^2) (P_2^- - P_2^+) - \\
\times \frac{Q}{V 2\pi} \frac{\text{sh} \gamma_0 x}{\gamma_0} S(x) + (K_x^{-1} - 1) \left[ \frac{d\overline{T}}{dx} \right]_{x=h=0} \delta_+(x + h) - \\
\times \left[ \frac{d\overline{T}}{dx} \right]_{x=h=0} \delta_-(x - h) - \frac{Q}{V 2\pi} \delta(x),
\]

where \( \beta^2 = k_y^{(1)} \eta^2 + x_1^2 \).