NOTATION

Here $p$, $r$, $t$, are pressure, coordinate, time; $h$, $r$, coordinate and time steps; $u$, $v$, grid functions; $q$, well yield; $r_0$, radius of well; $a$, $e$, $g$, system parameters; a superimposed circumflex denotes values of the grid functions for the next time layer.

LITERATURE CITED


DESCRIPTION OF THE STRENGTH OF POROUS BODIES ON THE BASIS OF PERCOLATION THEORY

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A method is proposed for estimating the strength of porous materials in destructive rupture on the basis of the percolation theory of regularly packed spheres. The results of calculation by this method are in good agreement with experimental data in the whole porosity range of the material.

Percolation theory [1, 2] is widely used to describe various probabilistic processes. There have been numerous studies of the elasticity of two- and three-dimensional percolation systems, for example [3-7]; it was noted in [4, 5] that the elasticity and conductivity problems for the volume elastic modulus of gel at the gel point belong to different universality classes. In [7], the method of reduction to an elementary cell was used to determine the moduli of the percolation systems, permitting considerable simplification of the calculations. An analogy between the mechanical characteristics and thermal conductivity of porous powder materials was made in [8]. In [9], percolation theory was used to describe the strength and rheological characteristics of disperse systems; however, the use of Bethe lattices limits the application of the given approach to transitions of sol-gel type. The percolational approach to the description of the strength of porous media was considered in [10]; the dependence of the relative strength $G$ (short-term resistance of the material in rupture referred to its maximum value) on the relative particle concentration $m/m_0 = (1 - \Pi)/(1 - \Pi_0)$ where $\Pi_0$ is the porosity of the system corresponding to the densest random packing of the particles, was derived. One advantage of this approach is the elimination of the traditional consideration of the strength using the concept of the "failure surface," which leads to complication in taking account of statistical inhomogeneities of the system, for example, macropores. The calculation method is based on an analogy between conduction and the strength of a percolation system, but this analogy is more explicit if the proportion of conducting point of the percolational system is identified with the mean relative coordination number $v = (Z + 1)/(Z_0 + 1)$, which characterizes the proportion of possible particle bonds realized on average in the given case.

An analytical expression from [11] is used to calculate the mean coordination number of a lattice of randomly packed spherical particles as a function of the porosity of the packing.
Fig. 1. Generalized form of inhomogeneous model: a) general view; b) model with interpenetrating components.

Fig. 2. Dependence of the relative strength of a body on its porosity: points) experimental data; curve) calculation by the method here proposed.

The coordination $Z_0$ of a lattice of randomly close-packed spheres (the porosity $\Pi_0$ is 0.39 according to [10]) is 7.28 according to Eq. (1).

To determine the percolation threshold of the given lattice, it is convenient to use the dependence

$$\psi = 0.15/(1 - \Pi_c),$$

which provides a good description of the influence of porosity on the percolation threshold of regular three-dimensional lattices [2].

It follows from Eqs. (1) and (2) that

$$\left(\Pi_c + 3 + \sqrt{\Pi_c^2 - 10\Pi_c + 9}/(2\Pi_c) + 1\right) = \frac{0.15}{1 - \Pi_c}. \quad (3)$$

The threshold lattice porosity $\Pi_c = 0.729$ is obtained from Eq. (3) by an iterative method for $\Pi_0 = 0.39$. Then, substituting this value of $\Pi_c$ into Eq. (1), the coordination number of the lattice $Z_c = 3.584$ is determined and hence the percolation threshold $\nu_c = 0.553$.

Knowing $\nu_c$, the relative strength close to the percolation threshold may be calculated using the scaling asymptote.

Far from the percolation threshold, the dependence of the lattice conductivity on its coordination number is used, taking account of the analogy between the strength and conductivity of percolational systems [10]

$$G(\nu) = 1 - \frac{2Z - 2}{Z - 2} (1 - \nu).$$

(4)