ONE APPROACH TO CONSTRUCTING A METHOD FOR DESIGNING
MODEL HEAT SHIELDS

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This paper proposes an engineering approach to construction of a method for designing multilayer heat shields that is based on iterative use of rigorous and simplified mathematical models and permits effective assembly synthesis on the basis of the condition that specified constraints on temperature conditions be satisfied.

The optimum-mass solution to the problem of designing a one-dimensional assemblage (stack) with specified structure that will be exposed to a high-temperature environment and is characterized by constraints on the temperature conditions in individual zones is the solution of a problem of the type:

\begin{align*}
M &= \sum_{j=1}^{n} \rho_{\text{var},j} h_{\text{var},j} \rightarrow \min, \\
T_{\text{con},i} &\leq \hat{T}_{\text{con},i}, \quad i = 1, m, \\
h_{\text{var},i} &\geq \bar{h}_j, \quad j = 1, n.
\end{align*}

Here the temperature conditions are described within the framework of the one-dimensional Fourier equation [1].

The penalty function method [2] is used to reduce problem (1)-(3) to the unconditional minimization problem

\begin{equation}
F = \sum_{j=1}^{n} \rho_{\text{var},j} h_{\text{var},j} + \sum_{i=1}^{m} a_i \max(0, T_{\text{con},i} - \hat{T}_{\text{con},i}) + \sum_{j=1}^{n} b_j \max(0, \bar{h}_j - h_{\text{var},j}) \rightarrow \min \tag{4}
\end{equation}

Various aspects of the development of methods for solution of this sort of problem were considered in [3, 4]. However, the above techniques have not found wide application in investigation of practical shield design problems. Considerably more frequent use is made of a simplified approach to assemblage synthesis based on seeking that combination of thicknesses for the individual layers (whose total number is m) which assures satisfaction of conditions having the form

\begin{equation}
\eta(h_{\text{var},1}, \ldots, h_{\text{var},m}) = T_{\text{con},i}(h_{\text{var},1}, \ldots, h_{\text{var},m}) - T_{\text{con},i} = 0, \quad i = 1, m. \tag{5}
\end{equation}

The present study is devoted to methodological problems pertaining to construction of solutions for problems of the type formulated above.

One possible iterative approach to solution of problem (5) (which we will call algorithm I) consists in performing a sequence of operations in each k-th iteration:

- formation of an initial approximation \( h_{\text{var},j}^{(k)} \) (\( j = 1, m \)) for the layer thicknesses sought;
- calculation of the functionals \( \eta_i^{(k)} \) and their partial derivatives \( \eta_{h_{\text{var},j}}^{(k)} \) with respect to the arguments \( h_{\text{var},j} \) (\( i, j = 1, m \)) using the heat conduction equation;
- determination of the layer thickness increments \( \Delta h_{\text{var},j}^{(k)} \) satisfying the system of linear algebraic equations

Fig. 1. Structure of search for problem solution. 1) Input of initial data; 2) initial formation of corrective links; 3) simplified mathematical model; 4) module for making decision as to whether to continue search for solution in inner loop; 5) rigorous mathematical model; 6) module for making decision as to whether to continue search for problem solution in outer loop; 7) current formation of corrective links; 8) exit from iterative process.

\[
\sum_{j=1}^{m} q_{h,i,j}^{(k)} \Delta h_{var,i}^{(k)} = -q_{i}^{(k)}, \quad i = \bar{1}, \bar{m},
\]

obtained by linearization of system of equations (5).

The transition from the k-th to the k + 1-th iteration is made with equations having the form

\[
h_{var,i}^{(k+1)} = h_{var,i}^{(k)} + \Delta h_{i}^{(k)}, \quad j = \bar{1}, \bar{m}, \quad k = 1, 2, \ldots;
\]

(7)

\[
\Delta h_{i}^{(k)} = \begin{cases} \Delta h_{var,i}^{(k)}, & \vert \Delta h_{var,i}^{(k)} \vert \leq \vert \Delta h_{var,i}^{(k)} \vert, \\ \Delta h_{var,i}^{(k)}, & \vert \Delta h_{var,i}^{(k)} \vert > \vert \Delta h_{var,i}^{(k)} \vert, \end{cases}
\]

(8)

\[
\Delta h_{var,i}^{(k)} = \beta h_{var,i}^{(k)} \text{sign}(\Delta h_{var,i}^{(k)}), \quad j = \bar{1}, \bar{m}.
\]

(9)

The iteration process is terminated at that iteration \( \ell \) in which the condition

\[
\vert q_{i}^{(\ell)} \vert \leq \varepsilon_{i}, \quad i = \bar{1}, \bar{m},
\]

is satisfied.

Specific calculations made with this algorithm will be given below. The effectiveness of algorithm 1 is to a considerable extent determined by the choice of initial approximation. Moreover, special emphasis must be placed on the fact that use of adequately fitting mathematical models (particularly those that take into account the dependence of thermo-physical properties on temperature and pressure, the nonlinearity of the boundary conditions, and the multidimensional nature of the problem) in this scheme often leads to expenditure of large amounts of computer time in order to obtain the solution sought.

When this sort of situation arises, one generally proceeds by utilizing simplified mathematical models of the process under investigation. However, alternatives must be sought if this approach does not yield a satisfactory solution to the problem (because of violation of restrictions on the permissible error in the solution found).

One effective way to overcome these difficulties was reported in [5], which describes investigation of mathematical models of radiative-convective heat transfer in high-temperature shock layers.

We can, utilizing this latter study, propose the following search structure (depicted in Fig. 1) for seeking a solution described by complex reentrant mathematical models (in [5], this reenterability was a product of the use of iterative methods for iteration processes with relatively slow convergence). The overall procedure for seeking a solution is