THE HEATING OF MATERIALS BY A CONCENTRATED ENERGY FLUX UNDER VOLUME ABSORPTION

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An analysis is represented for the process of heating a massive body by concentrated energy flux with volume absorption taken into account. The boundaries of applicability are determined for models of surface and volume absorption. The temperature fields are computed and simple analytical dependences are obtained for the temperature at the center of the heating spot.

The action of a concentrated energy flux (CEF) on a material results in the formation of a heat source with definite space-time characteristics on the surface and within the bulk of a body [1, 2]. This is a surface source for metals in the majority of cases since the degree of laser radiation penetration into a metal is $10^{-6}$ cm [1]. For a number of other CEF, for instance, for highly-energetic electron beams with 100 kV accelerating voltage and higher, the electrons penetrate into the metal to a depth of tens of μm and more [2]. Moreover, the action of laser radiation on a weakly absorbing medium results in the formation of a heat source decreasing exponentially with the growth of the depth (according to the Bouger law) in the bulk of a body. Consequently, the determination of whether the heat source being formed is surface or volume depends on a number of parameters of the problem including the radius of the heating spot, the action time, etc.

The knowledge of whether the heat source should be considered surface or volume in a specific situation of CEF action on a material is important for a number of technological processes including heat-treatment, welding, etc. [1, 2]. This situation was analyzed first in [3] for a normally distributed heat source, dropping exponentially in the bulk of the material. However, the analysis in [3] was performed only for the cases of small and large times of CEF action while intermediate time cases were not considered.

The purpose of this paper is to determine during what time intervals the surface or volume nature of the absorption of its energy in the material should be considered for a heat source. The deductions obtained can be carried over qualitatively to other energy distribution regularities over the body surface also.

The mathematical formulation of the problem includes a linear heat conduction equation in spatial form without heat losses from the surface and has the form

$$\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2} + \frac{\partial^2 T'}{\partial z'^2} = \frac{1}{a} \frac{\partial T'}{\partial t'} - \frac{Aq_0\alpha'}{\lambda V k} \exp \left[-\alpha' z' - k (x'^2 + y'^2)\right],$$

$$0 < z' < \infty, \quad \infty < x', \quad y' < \infty,$$

$$\frac{\partial T'}{\partial z'} \bigg|_{z'=0} = 0, \quad T' (x', y', z', 0) = T' (\infty, y', z', t') = T' (x', \infty, z', t') = T'(x', y', \infty, t') = T_e.$$  \hspace{1cm} (1)

Introducing the dimensionless variables in the following manner

$$y = \sqrt{k} y', \quad z = \sqrt{k} z', \quad x = \sqrt{k} x', \quad t = k a t', \quad \alpha = \frac{\alpha'}{V k},$$

$$T = \frac{\lambda}{Aq_0} (T' - T_0)$$

and using the symmetry of the problem, i.e., no heat losses from the surface \( \partial T/\partial z|_0 = 0 \), we obtain a solution of the system (1) by using the Green's function

\[
T(x, y, z, t) = \frac{\alpha}{16\pi \sqrt{\pi t}} \int_0^t (t - \tau)^{-3/2} \int_{-\infty}^\infty dx' \int_{-\infty}^\infty dy' \int_{-\infty}^\infty dz' \times
\exp \left\{ -\alpha |x'| - (x'^2 + y'^2) - \left[ \frac{(x-x')^2 + (y-y')^2 + (z-z')^2}{4(t-\tau)} \right] \right\}.
\]

The integrals over \( x', y' \) and \( z' \) are calculated yielding

\[
T(x, y, z, t) = \frac{\alpha}{8} \int_0^{1/T} \frac{\xi dz}{\xi^2 + 1} \exp \left( \frac{\alpha \xi^3}{4} - \frac{\rho^2}{1 + \xi^2} \right) \times
\times \left\{ \text{ch}(-\alpha z) + \frac{1}{2} \left[ \exp(-\alpha z) \text{erf} \left( \frac{z}{\xi} - \frac{\alpha \xi^3}{2} \right) - \exp(\alpha z) \text{erf} \left( \frac{z}{\xi} + \frac{\alpha \xi^3}{2} \right) \right] \right\}.
\]

(4)

where \( \text{erf}(x) \) is the error integral and \( \rho^2 = x^2 + y^2 \).

The expression simplifies considerably at the center of the heating spot

\[
T(0, 0, 0, t) = \frac{\alpha}{4} \int_0^{1/T} \frac{\xi dz}{\xi^2 + 1} \exp \left( \frac{\alpha \xi^3}{4} \right) \text{erfc} \left( \frac{\sqrt{\alpha \xi^3}}{2} \right).
\]

(5)