A model is proposed for the motion of a gas–liquid system in tubes. The theoretical and experimental results are in good agreement.

The complexity and multiplicity of the flow systems leads to considerable difficulties in studying the motion of gas–liquid systems in tubes, both in conducting the experiments and in constructing the mathematical models. Nevertheless, there have already been many experimental and theoretical investigations of the motion of gas–liquid flows in tubes (see [1-5], etc.), in which processes at pressures below the saturation pressure are mainly considered.

However, gas–liquid flows at pressures above or close to the saturation pressure are investigated as homogeneous systems, as a rule, on the assumption that transitions from one state to another occur instantaneously in the equilibrium thermodynamic theory of phase transitions.

However, according to the data of [6], the formation of a new phase occurs not instantaneously over the whole volume but rather takes the form of local fluctuations passing beyond the limits of a single aggregate state. Nuclei of new phase (gas bubbles) are "heterophase" and it is assumed, in accordance with the results of [6], that in the region above and especially close to the saturation pressure the system is not completely homogeneous. The "heterophase" system may be both in equilibrium and in a nonequilibrium state. As a rule, in the given conditions, the dispersed gas is uniformly distributed over the liquid volume.

Experiment shows [7, 8] that in a point volume the pressure level and its rate of change influence the formation of micronuclei. In connection with this, experiments are conducted to determine the moment of appearance of micronuclei of the gas phase. In a container connected to a press, a gasified liquid is prepared; it consists of transformer oil and carbon dioxide at the saturation pressure (0.04 MPa). Then the pressure is increased systematically to 0.25 MPa, i.e., considerably above the saturation pressure. Then the pressure drops systematically reduced at a definite rate to different levels above the saturation pressure. Analysis of the experimental results shows that, beginning at some value (P = 0.17 MPa), the pressure increases more rapidly (Fig. 1); this may be due to the formation of micronuclei of gas phase.

The presence of micronuclei in the system leads to a significant dependence of the density on the pressure and the rate of change in pressure, and probably is responsible for the decrease in values of the rheological parameters observed experimentally in [9, 10].

Taking account of the foregoing, a model may be proposed for the motion of gas–liquid systems in tubes at pressures above or close to the saturation pressure.

1. The system of differential equations for the motion of the gas–liquid medium is written as in [11]
Fig. 1. Pressure variation over time at various levels above the saturation pressure. P, 10 MPa; t, sec.

\[
\frac{\partial (f \rho W)}{\partial t} + \frac{\partial (f \rho W^2)}{\partial x} = -f \frac{\partial P}{\partial x} + \chi, \tag{1}
\]

After some manipulations, Eq. (1) may be written in the form

\[
\frac{\partial W}{\partial t} + \frac{1}{2} \frac{\partial W^2}{\partial x} = -\frac{1}{\rho_0} \frac{\partial P}{\partial x} + \frac{\chi}{\rho_0} \tau(W), \tag{2}
\]

Suppose that the cross-sectional area of the tube depends on the pressure according to the law

\[
f = f_0 \left(1 + a \frac{P - P_0}{E}\right), \tag{3}
\]

Since it is assumed that the formation of new phase is not instantaneous, a relation between the density and pressure may be used [12]

\[
\frac{\rho_0}{k_0^0} \left( P - P_0 + \theta_p \frac{dP}{dt} \right) = \rho - \rho_0 + \frac{k_0^m}{k_0^0} \theta_p \frac{dP}{dt}. \tag{4}
\]

The dependence of the stress on the velocity is specified as for a viscous liquid [11]

\[
\tau = -\frac{4\mu}{R} W. \tag{5}
\]

In the presence of gas inclusions distributed over the whole volume of the liquid, the viscosity of the system may be written analogously [13]

\[
\mu = \mu_0 (1 + \gamma C). \tag{6}
\]

The value of \(\gamma\) may be both positive (in [13], a value of 2.5 was adopted for spherical solids) and negative (for gas inclusions).