3D Multivalued Travel Time and Amplitude Maps

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Abstract—An algorithm for computing multivalued maps for travel time, amplitude and any other ray related variable in 3D smooth velocity models is presented. It is based on the construction of successive isochrons by tracing a uniformly dense discrete set of rays by fixed travel-time steps. Ray tracing is based on Hamiltonian formulation and includes computation of paraxial matrices. A ray density criterion ensures uniform ray density along isochrons over the entire ray field including caustics. Applications to complex models are shown.

Key words: 3D Ray tracing, travel time, amplitude, Hamiltonian, Lagrangian manifold, 3D Overthrust model.

1. Introduction

The current interest in 3D seismic imaging has considerably increased the importance of ray tracing methods in wave field computations. Among seismic modelling methods, ray tracing methods provide a reasonable compromise between accuracy and computational efficiency (Hanyga and Helle, 1994) while the alternative methods such as finite differences (Aminzadeh et al., 1994a,b; Aminzadeh et al., 1995) and spectral methods require substantial computing power.

Linear inversion based on the ray + Born (Lambarè et al., 1992;Forgues et al., 1994; Ettrich and Gajewski, 1995) or ray + Rytov approximation requires repeated evaluation of amplitudes, travel times and other ray related variables for each ray connecting the source/receiver and the scatterers. The background velocity model must be inhomogeneous for general applications. However, in many cases it can be assumed smooth at least in a first step (Versteeg, 1991). Model discontinuities can be reconstructed by linear inversion of data residuals with respect to the data calculated for the background. A forward operator for such a linear inversion

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is given by the Born approximation, provided an efficient algorithm for numerical computation of Green's functions in the background model is available. We address here the problem of efficient determination of multivalued 3D maps for travel time, amplitude or any other ray related variables throughout the target zone from any shot and receiver position at the surface and, indirectly, for numerical evaluation of asymptotic Green's functions by ray tracing.

A significant breakthrough in the travel-time computation was achieved through the finite difference (FD) calculation of the first-arrival travel time (Vidale, 1988; Podvin and Lecomte, 1991). However, FD computation of travel times leads to poor imaging in complex media (Geoltrain and Brac, 1991), due to unreliable amplitude information. Simultaneous computation of travel times and amplitudes is possible by dynamic ray tracing (Farra and Madariaga, 1987; Farra, 1993) of a densely sampled ray field and evaluation of travel times and amplitudes at given points by interpolation. The ray density must be controlled in order to ensure accuracy as well as computational efficiency of the algorithm.

For the 2D case, various methods have been proposed (Lambare et al., 1992; Forgues et al., 1994; Vinje et al., 1992, 1993a; Sun, 1992). The wave front construction method proposed by Vinje et al. appears reasonably efficient, even in the 3D case (Vinje et al., 1993b). It is based on subdivision of the ray field into elementary cells defined by adjacent rays and successive isochrons. A criterion controls the ray density over the isochrons and new rays are added by interpolation when necessary. In Vinje et al. (1993a) such a criterion was formulated in terms of the metric distance between adjacent rays.

The method was improved in 2D successively by Sun (1992) and Lambare et al. (1996) by introducing more efficient ray density criteria. The ray density criterion of Lambare et al. (1996) ensures a uniform sampling of the ray field, in particular in caustic regions, while the criteria of Vinje et al. (1993a), Vinje et al. (1993b) and Sun (1992) resulted in a drastic undersampling in these zones (Lambare et al., 1996).

In Lambare et al. (1996) ray equations are recast in the Hamiltonian form (Chapman, 1985). In the Hamiltonian formulation, rays in the configuration space \((x)\) are replaced by bicharacteristics in the phase space \((x,p)\), where \(p\) denotes the slowness vector. The set of bicharacteristics associated with a source spans a regular Lagrangian manifold \(\Lambda\) in the phase space (Weinstein, 1979). Tangent planes to the Lagrangian manifold \(\Lambda\) can be determined by paraxial ray tracing. The ray density criterion of Lambare et al. (1996) is based on the \(x\) and \(p\) curvatures of isochrons in the phase space.

We present in this paper an extension to the 3D case of the 2D algorithm (Lambare et al., 1996). We first recall ray theory, Hamiltonian equations for ray tracing and paraxial ray tracing. We then describe the numerical scheme for the wave front construction method in 3D. Finally we present some examples.