Birefringence Parameters: Indicators of Anisotropic Symmetry Systems

IVAN DE ARAÚJO SIMÕES-FILHO

Abstract—The difference in travel times between split shear waves (travel-time splitting) in anisotropic media depends on nine combinations of the density normalized elastic parameters (the birefringence parameters). These combinations are all zero in isotropic media, where there is no shear-wave splitting. The number of nonzero birefringence parameters increases with decreasing symmetry elements in the medium: from one in cubic media to nine in triclinic media.

The birefringence parameters may be recovered from travel-time splitting observations. Their azimuthal behavior may then be interpreted in terms of crack orientation (strike and dip directions).

Key words: Birefringence parameters, weak elastic anisotropy, shear-wave travel-time delay, anisotropic symmetry systems.

1. Introduction

Anisotropic media can have symmetry elements (planes and/or axes of symmetry) which are grouped into symmetry systems. These systems are classified according to the number and relative orientations of the symmetry elements. The number of independent elastic parameters needed to describe the elastic properties of the medium is constrained by the symmetry system: the higher the symmetry, the lower the number of independent elastic parameters. A description of the symmetry systems and their relations to the number of independent elastic parameters can be found in CRAMPIN (1981) or HELBIG (1994).

The importance of the description of the symmetry system lies in the fact that it is a result of the internal structure of the minerals and/or rocks that constitute the medium. Given an elastic equivalent medium, its elastic parameters could be inverted for parameters of a microscopic model, e.g., a crack model or a thin layer model such as those proposed by HUDSON (1980, 1981) or SCHOENBERG and MUIR (1989). The information obtained in this way would be smaller in scale than the resolution of the seismic method.

1 UNICAMP/IG/AGP, C.P. 6152, 13084-100 Campinas-SP, Brasil.
This paper demonstrates that, within the framework of first-order perturbation theory, only nine combinations of the density normalized elastic parameters control the travel-time delay between split shear waves in weakly anisotropic media. How these combinations (the birefringence parameters) can be used to identify most anisotropic symmetry systems is also discussed.

This identification is important, for instance, to constrain the number of independent density normalized elastic parameters sought in travel-time inversion codes, based on travel times of individual body waves (qP, qS1 and qS2).

2. Equation for the Travel-time Delay

Efficient travel-time computations can be obtained by a combination of ray methods and perturbation theory, particularly when ray computations are performed in an isotropic (unperturbed) background medium. Travel-time perturbation methods are then used to estimate the travel times in a perturbed anisotropic medium whose density normalized elastic parameters are close to the parameters of the isotropic background. The travel-time perturbations are computed by integration of the perturbations of density normalized elastic parameters along the ray in the isotropic background, with the travel time \( \tau \) along the ray as the integration step. This has a very important practical application, as it is much simpler to perform ray computations in isotropic than in anisotropic media.

Čerňený (1982) and Hanyga (1982) derived the equation for travel-time perturbations of seismic body waves in inhomogeneous anisotropic media. These equations are valid for \( qP \) waves without any restriction on the background medium, however their validity to \( qS \) waves is restricted to an anisotropic nondegenerate background.

This restriction was removed with the use of degenerate perturbation theory. Jech and Pšenčík (1989) derived the formulae to obtain the travel-time perturbation for both \( qS \) waves, when the unperturbed (background) medium is isotropic. They demonstrate that the travel-time delay between split shear waves in anisotropic media may be computed by the following equation:

\[
|\Delta T_{qS1-qS2}| = \frac{1}{2} \int [(B_{11} - B_{22})^2 + 4B_{12}^2]^{1/2} d\tau,
\]

where

\[
B_{mn} = \delta a_{ijkl} P_i P_j e_{(m)} e_{(n)}.
\]

Here \( p_i \) are the Cartesian components of the slowness vector in the background medium and \( e_{(m)} \) and \( e_{(n)} \) are any two mutually perpendicular unit vectors located in the plane normal to \( p \). The perturbations of the density normalized elastic parameters \( \delta a_{ijkl} \) are the differences between the elements of the fourth-order tensor.