SH-Waves from a Finite Line Source

By S. S. De

Summary – The displacements and shearing stresses in an infinite medium are obtained for a finite line source, taken as the limit of a prolate ellipsoid with the shearing stresses prescribed on its surface. The results have been extended to the case of a semi-infinite medium by the method of images.

1. Introduction

The problem of the propagation of elastic waves from a finite line source has its application in seismology. Heelan [3] attempted this problem by considering an infinite narrow cylindrical cavity with the prescribed stresses applied to a finite length of its wall. Jordan [4] reconsidered the same problem. Mitra [6] considered a finite SH line source due to the presence of crossradial body forces and obtained the surface displacement in closed form. Recently this problem has been attacked by Hazebroek [2] from a different point of view. He considered the line source as the limit of an elongated ellipsoid of revolution, the length of the minor axis of which tends to zero. The only prescribed stress on the wall of the cavity is the normal stress, the others being assumed to be zero. Spheroidal wave functions have been used in solving the problem. In this paper a problem has been considered similar to that of Hazebroek with the difference that the shearing stress instead of the normal stress is prescribed on a cavity in the infinite medium. For simplicity, cylindrical symmetry has been assumed. On making the minor axis tend to zero the solution for a finite line source emitting shear pulses is obtained. The solution has been extended to the case of a semi-infinite medium by using the method of images; this is possible due to the fact that only SH motion is considered.

2. Problem

The line source has been considered as the limit of a prolate ellipsoid the major axis of which is along the z-axis in the infinite isotropic elastic medium. Spheroidal coordinates (ξ, η, φ) have been used and the only non-zero stress acting on the wall of the cavity is the shearing stress τ_φφ, in the direction of φ increasing, which is a pre-

1) Department of Mathematics Bengal Engineering College, Howrah-3, West Bengal, India.
2) Numbers in brackets refer to References, page 59.
scribed function of time and independent of $\eta$; the response has been calculated for an initial disturbance represented by $\delta(t)$. The problem has been tended to the case of an elastic half-space and for the case of an initial disturbance represented by Heavyside's step-function.

3. Equations in terms of spheroidal coordinates

The displacement vector $\vec{d}$ in an elastic medium under no body forces satisfies the vector equation

$$\mu \nabla \times \nabla \times \vec{d} - (\lambda + 2\mu) \nabla \nabla \cdot \vec{d} + \frac{\partial^2 \vec{d}}{\partial t^2} = 0. \tag{3.1}$$

Firstly, it is assumed that $\vec{d} \propto e^{i\omega t}$, so that

$$\mu \nabla \times \nabla \times \vec{d} - (\lambda + 2\mu) \nabla \nabla \cdot \vec{d} - \rho \omega^2 \vec{d} = 0. \tag{3.2}$$

If $\psi$ satisfies the equation of the type

$$\nabla^2 \psi + m^2 \psi = 0, \quad (m = \text{a constant}).$$

Stratton [7] has shown that a set of solutions of equation (3.2) is

$$\vec{a} = v \times \vec{a}, \quad \vec{v} = \nabla \times \vec{a}, \quad \vec{m} = 1 - \frac{\nabla \times \vec{M}}{m} \quad \tag{3.3}$$

where $\vec{a}$ is any unit vector.

For the present problem, the set

$$\vec{L} = \nabla \psi_0, \quad \vec{M} = \nabla \times \vec{d} \psi_0 \quad \tag{3.4}$$

will suffice, where $\psi_0, \psi$, satisfy the equations

$$\nabla^2 \psi_0 + \frac{\rho \omega^2}{\lambda + 2\mu} \psi_0 = 0 \quad \tag{3.5}$$

$$\nabla^2 \psi + \frac{\rho \omega^2}{\mu} \psi = 0.$$ 

It will be seen later that $\psi_0 \equiv 0$. Then the displacement vector can be expressed as

$$\vec{d} = \vec{L} + \vec{M}. \quad \tag{3.6}$$

Let

$$a^2 = \frac{\lambda + 2\mu}{\rho}, \quad b^2 = \frac{\mu}{\rho},$$

$$h = \frac{\omega}{a}, \quad \kappa = \frac{\omega}{b}. \quad \tag{3.7}$$