Refraction of Elastic Waves into a Medium of Lower Velocity – Pseudospherical Waves

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Summary – An irregular wave group (here called pseudospherical), the existence of which is connected with the velocity boundary at which the velocity decreases discontinuously, is investigated. A schlieren modelling device was chosen for a model investigation of this wave since it permits the investigation of wave fields inside the measured models. The model consisted of two layers of transparent gels, the source lay in the layer of higher velocity. The measurements have shown that an irregular wave in the layer of lower velocity exists only in a certain region along the boundary; its wave front has a spherical form and its intensity decreases rapidly with increasing distance of the source from the boundary. The wave always comes only after the regular refracted wave which conforms with the ray theory. These properties correspond to the properties of a wave first described by OTT [1]³) and BREKHOVSKYKH [2]. In the conclusion of the present paper the possibilities of recording pseudospherical waves in seismology are outlined.

I. Introduction

During the last three years schlieren measurements on layered seismic models have been carried out at the Geophysical Institute of the Czechoslovak Academy of Sciences. In studying elastic waves propagating in low-velocity channels a certain irregular wave was observed in the records obtained (KOZÁK [6]), which could not be explained by means of the ray theory. It was found that the existence of this wave is not directly related to the low-velocity channel but to the boundary at which the velocity decreases discontinuously. Therefore, more detailed model measurements on two-layer models have been performed with the object of further studying the wave in question and comparing the results of measurements with some theoretical conclusions. The schlieren modelling technique which enables the investigation of the wave field inside the model was chosen for the model measurements. In keeping with the principles of the schlieren recording, the kinematic and partially also the dynamic properties of an irregular wave group were investigated in a plate model consisting of two kinds of transparent gels with a different velocity of compressional waves. The

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model was placed in a special model vessel (cuvette). Because of the irregular character and the spherical wave front of the wave group under study, it will be called a pseudo-spherical wave in this paper.

2. Theoretical considerations

The reflection and the refraction of a spherical wave at a plane boundary between two homogeneous elastic media have already been investigated in detail by various authors (see e.g. OTT [1], BREKHOVSKIKH [2], CAGNIARD [3], EWING, JARDETZKY and PRESS [4], ČERVENÝ and RAVINDRA [5]); consequently, only the results directly concerning the subject of the present paper will be briefly summarized here. For simplicity both media will be considered liquid. Let a point source of harmonic waves lie at a distance $h$ from the boundary. The medium in which the source lies will be called the first medium. The velocity of compressional waves and the density in the first medium will be designated $\alpha_1, \rho_1$, respectively; the same quantities in the second medium will be $\alpha_2, \rho_2$. Further, the designation $n=\alpha_1/\alpha_2$ (refractive index) and $\eta=\rho_1/\rho_2$ will be introduced. Only the case $n>1$ will be dealt with (i.e., the velocity in the medium with the source is higher than the velocity in the second medium). Cylindrical coordinates $r, z, \varphi$ are introduced in such a way that the source lies on the $z$-axis (at the point $S$ with the coordinates $r=0, z=h$) and the boundary coincides with the plane $z=0$. For reasons of symmetry the wave field does not depend on the coordinate $\varphi$. The potential of a spherical wave generated by the source $\Phi_0$ is considered here in a simple form

$$\Phi_0 = \frac{e^{ikR_0}}{R_0} e^{-i\omega t},$$

where $t=$time, $k=\omega/\alpha_1$, $\omega=2\pi f$ (f = frequency), $R_0=\sqrt{r^2+(z-h)^2}$.

If the wave described by potential (1) incidents upon the boundary, reflected waves (in the first medium) and refracted waves (in the second medium) originate. Let us deal with the waves in the second medium. BREKHOVSKIKH [2] has deduced theoretically that two waves – one regular and one irregular (see Fig. 1) – arrive to the point $P[r, H]$ which does not lie too far from the boundary:

a) Regular wave corresponding to the ray theory. It will be called a regular refracted wave in this paper. The energy of this wave is transferred along the ray $SMP$ (see Fig. 1). The angle of incidence of the ray $\theta_1$ and the angle of refraction $\varphi_2$ are connected by Snell’s law ($\sin \theta_2=(\alpha_2/\alpha_1) \sin \theta_1$). At great distances from the source ($\theta_1 \sim \pi/2$), $\sin \theta_2$ will change only slightly ($\sin \theta_2 \sim \varphi_2/\alpha_1$). Therefore, the wave front of a regular refracted wave will be approximately conical in the neighbourhood of the boundary and it will form an angle with the boundary the sine of which will be close to $\alpha_2/\alpha_1$. (In the plane $r z$ the wave front will be approximately a straight line.) At greater distances from the boundary the wave front will be curved, of course.

b) Irregular wave (here called pseudospherical) which does not exist in the ray