Rayleigh Waves in Granular Medium

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Summary – The wave velocity equation in the form of a ninth order determinantal expression is derived appropriate to Rayleigh type waves in a granular half-space supporting a different granular layer. The classical frequency equation when both media are elastic has been deduced as a particular case by limiting process.

1. Introduction

The study of granular medium in recent times has been necessiated by its possible applications in soil mechanics, geophysical prospecting, mining engineering etc. References of investigations, both theoretical and experimental outlining the development of the subject from the mid-thirties has been given by Paria [4]2). The present paper however is based on the Dynamics of Granular Media as propounded by Oshima [2, 3]. The medium under consideration is a discontinuous one such as composed of numerous large or small grains. ‘Unlike a continuous body, each element or grain cannot only translate but also rotate about its centre of gravity. This motion is the characteristic of the medium and has an important effect upon the equations of motion to produce internal friction. It is assumed that the medium contains so many grains that they will never be separated from each other during the deformation and that the grain has perfect elasticity’. Propagation of Rayleigh type waves in such a half space overlain by a different granular layer is considered in this paper. The wave velocity equation has been derived in the form of a ninth order determinant, generalizing the classical frequency equation for two elastic media. The roots of this equation are in general complex and the imaginary part of an appropriate root measures the attenuation of the waves.

2. Summary of the field equations

The state of deformation in the granular medium is described by the displacement vector \( \vec{u}(u, v, w) \) of the centre of gravity of a grain and the rotation vector \( \vec{\xi}(\xi, \eta, \zeta) \) of the grain about its centre of gravity. There exist a stress tensor \( \tau_{ij} \) and a secondary

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2) Numbers in brackets refer to References, page 22.
stress tensor (or stress couple) $M_{ij}$ having components

\[
\begin{pmatrix}
\tau_{xx} & \tau_{yz} & \tau_{zx} \\
\tau_{xy} & \tau_{yy} & \tau_{zy} \\
\tau_{xz} & \tau_{yz} & \tau_{zz}
\end{pmatrix}
\]

and

\[
\begin{pmatrix}
M_{xx} & M_{yx} & M_{zx} \\
M_{xy} & M_{yy} & M_{zy} \\
M_{xz} & M_{yz} & M_{zz}
\end{pmatrix}
\]  

respectively. $\tau_{xx}$, $\tau_{xy}$ and $\tau_{xz}$ are the components of the resultant force acting on a surface element of a unit area perpendicular to the x-axis and $\tau_{yx}$, ... are similarly defined. Also $M_{xx}$, $M_{xy}$, $M_{xz}$ etc. are the components of the resultant couple acting on a surface element. The tensors are non-symmetric, i.e.

\[\tau_{ij} \neq \tau_{ji}, \quad M_{ij} \neq M_{ji}.\]

$\tau_{ij}$ can be expressed as the sum of a symmetrical and an antisymmetrical tensor as

\[\tau_{ij} = \sigma_{ij} + \sigma'_{ij},\]

where

\[\sigma_{ij} = \frac{1}{2} (\tau_{ij} + \tau_{ji}),\]

\[\sigma'_{ij} = \frac{1}{2} (\tau_{ij} - \tau_{ji}).\]

The symmetric tensor $\sigma_{ij} = \sigma_{ji}$ is related to the symmetric strain tensor

\[e_{ij} = e_{ji} = \frac{1}{2} (u_{i,j} + u_{j,i})\]

by the Hooke's Law and thus in isotropic medium we have

\[
\begin{align*}
\sigma_{xx} &= \lambda \varepsilon + 2\mu \varepsilon_{xx} & \sigma_{yy} &= 2\mu \varepsilon_{yy} \\
\sigma_{yy} &= \lambda \varepsilon + 2\mu \varepsilon_{yy} & \sigma_{zz} &= 2\mu \varepsilon_{zz} \\
\sigma_{zz} &= \lambda \varepsilon + 2\mu \varepsilon_{zz} & \sigma_{xy} &= 2\mu \varepsilon_{xy},
\end{align*}
\]

where

\[\varepsilon = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}\]

and $\lambda, \mu$ are Lamé's constants. As for the anti-symmetric stresses $\sigma'_{ij}$, we have,

\[
\sigma'_{yx} = -A \frac{\partial \xi}{\partial t}, \quad \sigma'_{xy} = -A \frac{\partial \eta}{\partial t}, \quad \sigma'_{yz} = -A \frac{\partial \xi}{\partial t}, \quad \sigma'_{xz} = \sigma'_{yz} = \sigma'_{zx} = 0,
\]

\[
\sigma'_{xx} = \sigma'_{yy} = \sigma'_{zz} = 0,
\]